UNIVERSIDADE FEDERAL DA GRANDE DOURADOS
Cálculo 2 — Avaliação P1
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Eng. de Computação 30/09/2026

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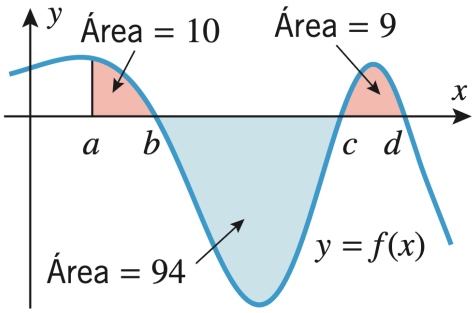
Nota	

Aluno(a): .....

Todas as respostas devem ser justificadas.

1. (2 pts) Determine o valor da intergal $\int_a^d f(x) + 2x - 1 \, dx$, onde o gráfico de f é dado abaixo.



2. (2 pts) Calcule a derivada de $f(x) = \int_{2x}^{3x} \frac{t^2 - 1}{t^2 + 1} \, dt$.

3. (2 pts) Encontre a primitiva geral de $f(x) = (x + 1)\sqrt{2x + x^2}$.

4. (2 pts) Determine o valor da integral definida $\int_0^{1/2} x \operatorname{sen}(\pi x) \, dx$.

5. (2 pts) Determine se a integral $\int_{-\infty}^{0} \frac{1}{3 - 4x} \, dx$ é convergente ou divergente e calcule seu valor se possível.

Boa Prova!

Solução

① $\int_a^d f(x)+2x-1\,dx = \int_a^d f(x)\,dx + \int_a^d 2x-1\,dx = 10-94+9+\left(x^2-x\Big|_a^d\right)$

$= -75 + d^2 - d - a^2 + a.$

② $\int_{2x}^{3x} \frac{t^2-1}{t^2+1}\,dt = \int_{2x}^{0} \frac{t^2-1}{t^2+1}\,dt + \int_0^{3x} \frac{t^2-1}{t^2+1}\,dt = \underbrace{-\int_0^{2x} \frac{t^2-1}{t^2+1}\,dt}_{F(x)} + \underbrace{\int_0^{3x} \frac{t^2-1}{t^2+1}\,dt}_{G(x)}$

Pela regra da cadeia e o TFC:

$F'(x) = -\frac{(2x)^2-1}{(2x)^2+1}\cdot 2 = -\frac{8x^2-2}{4x^2+1}$

$G'(x) = \frac{(3x)^2-1}{(3x)^2+1}\cdot 3 = \frac{27x^2-3}{9x^2+1}$

Portanto,

$$f'(x) = \frac{27x^2-3}{9x^2+1} - \frac{8x^2-2}{4x^2+1}.$$

③ Tomando $u = 2x+x^2$, temos $du = (2+2x)\,dx = 2(x+1)\,dx$

$\Rightarrow (x+1)\,dx = \frac{1}{2}du$. Assim,

$$\int (x+1)\sqrt{2x+x^2}\,dx = \int \sqrt{u}\cdot\frac{1}{2}\,du = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C$$

$$= \frac{1}{3}(2x+x^2)^{3/2}+C.$$

④ Integrando por partes:

$u = x \Rightarrow du = dx$

$dv = \operatorname{sen}(\pi x)\,dx \Rightarrow v = -\frac{\cos(\pi x)}{\pi}$

$$\therefore \int x\,\mathrm{sen}(\pi x)\,dx = -\frac{x\cos(\pi x)}{\pi} - \int -\frac{\cos(\pi x)}{\pi}\,dx$$

$$= -\frac{x\cos(\pi x)}{\pi} + \frac{1}{\pi}\int \cos(\pi x)\,dx = -\frac{x\cos(\pi x)}{\pi} + \frac{\mathrm{sen}(\pi x)}{\pi^2} + C$$

Portanto,

$$\int_0^{1/2} x\,\mathrm{sen}(\pi x)\,dx = -\frac{x\cos(\pi x)}{\pi} + \frac{\mathrm{sen}(\pi x)}{\pi^2}\Bigg|_0^{1/2}$$

$$= -\frac{\frac{1}{2}\cos\left(\frac{\pi}{2}\right)}{\pi} + \frac{\mathrm{sen}\left(\frac{\pi}{2}\right)}{\pi^2} + \frac{0\cos(0\pi)}{\pi} - \frac{\mathrm{sen}(0\pi)}{\pi^2} = \frac{1}{\pi^2}$$

(0, 0, 0)

⑤ $$\int_{-\infty}^{0} \frac{1}{3-4x}\,dx = \lim_{t\to-\infty}\int_t^0 \frac{1}{3-4x}\,dx$$

Calculando a primitiva:

$$\int \frac{1}{3-4x}\,dx = -\frac{1}{4}\int \frac{1}{u}\,du = -\frac{1}{4}\ln|u| + C = -\frac{1}{4}\ln|3-4x| + C$$

$(u = 3-4x \Rightarrow du = -4\,dx)$

Logo,

$$\int_t^0 \frac{1}{3-4x}\,dx = -\frac{\ln|3-4x|}{4}\Bigg|_t^0 = -\frac{\ln|3|}{4} + \frac{\ln|3-4t|}{4} = -\frac{\ln 3}{4} + \frac{\ln(3-4t)}{4}$$

$(t<0 \Rightarrow -4t>0)$

Assim,

$$\int_{-\infty}^{0} \frac{1}{3-4x}\,dx = \lim_{t\to-\infty}\left[-\frac{\ln 3}{4} + \frac{\ln(3-4t)}{4}\right] = +\infty$$

$(t\to-\infty \Rightarrow 3-4t \to +\infty \Rightarrow \ln(3-4t)\to+\infty)$.

Portanto, a integral é divergente.