



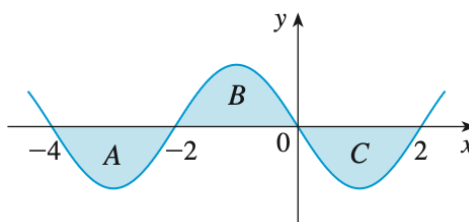
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2	
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Nota	

Aluno(a):.....

Todas as respostas devem ser justificadas.

1. (2 pts) Encontre todas as  $f$  tais que  $f'(x) = \frac{5}{2}x^3 - \frac{2}{x^2}$ .

2. (2 pts) Determine o valor da integral  $\int_{-4}^2 f(x) + 3x^2 - 1 \, dx$  sabendo que a área de cada região é 2.



3. (2 pts) Escreva como uma única integral na forma  $\int_a^b f(x) \, dx$ :

$$\int_{-3}^3 f(x) \, dx + \int_3^7 f(x) \, dx - \int_{-3}^{-2} f(x) \, dx$$

4. (2 pts) Calcule a integral  $\int_9^1 \frac{t-1}{\sqrt{t}} \, dt$ .

5. (2 pts) Calcule a área da região delimitada pelas curvas  $x = 1 - y^2$  e  $x = y^2 - 1$ .

## Solução

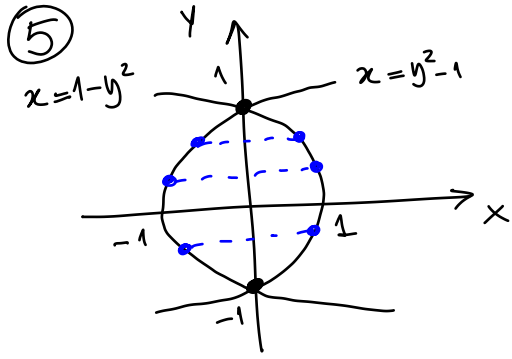
① Calculando a primitiva:

$$\begin{aligned}f'(x) = \frac{5}{2}x^3 - 2x^{-2} &\Rightarrow f(x) = \frac{5}{2} \cdot \frac{x^4}{4} - 2 \cdot \frac{x^{-1}}{-1} + C \\ &\Rightarrow f(x) = \frac{5}{8}x^4 + \frac{2}{x} + C.\end{aligned}$$

$$\begin{aligned}② \int_{-4}^2 f(x) + 3x^2 - 1 \, dx &= \int_{-4}^2 f(x) + \int_{-4}^2 3x^2 - 1 \, dx \\ &= -2 + \left( x^3 - x \Big|_{-4}^2 \right) = -2 + (8 - 2) - (-64 + 4) = 64.\end{aligned}$$

$$\begin{aligned}③ \int_{-3}^3 f(x) \, dx + \int_3^7 f(x) \, dx - \int_{-3}^{-2} f(x) \, dx \\ &= \int_{-3}^7 f(x) \, dx - \int_{-3}^{-2} f(x) \, dx = \int_{-3}^7 f(x) + \int_{-2}^{-3} f(x) \, dx \\ &= \int_{-2}^{-3} f(x) \, dx + \int_{-3}^7 f(x) \, dx = \int_{-2}^7 f(x) \, dx.\end{aligned}$$

$$\begin{aligned}④ \int_9^1 \frac{t-1}{\sqrt{t}} \, dt &= \int_9^1 \frac{t}{\sqrt{t}} - \frac{1}{\sqrt{t}} \, dt = \int_9^1 t^{1/2} - t^{-1/2} \, dt \\ &= \frac{2}{3}t^{3/2} - 2t^{1/2} \Big|_9^1 = \frac{2}{3} \cdot 1 - 2 \cdot 1 - \left( \frac{2}{3} \cdot 9^{3/2} - 2 \cdot 9^{1/2} \right) \\ &= \frac{2}{3} - 2 - \left( \frac{2}{3} \cdot 27 - 2 \cdot 3 \right) = \frac{2}{3} - 2 - 18 + 6 = \frac{2 - 6 - 54 + 18}{3} = -\frac{40}{3}.\end{aligned}$$



Intersec $\bar{a}$ o:

$$1 - y^2 = y^2 - 1 \Rightarrow 2y^2 = 2 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$$

$$\therefore A = \int_{-1}^1 (1 - y^2) - (y^2 - 1) dy = \int_{-1}^1 -2y^2 + 2 dy$$

$$= 2 \int_{-1}^1 -y^2 + 1 dy = 2 \left( -\frac{y^3}{3} + y \Big|_{-1}^1 \right)$$

$$= 2 \left[ -\frac{1}{3} + 1 + \frac{(-1)}{3} - (-1) \right] = 2 \cdot \left[ -\frac{2}{3} + 2 \right] = 2 \cdot \frac{4}{3} = \frac{8}{3}$$