1	
2	
3	
4	
5	
Nota	

UNIVERSIDADE FEDERAL DA GRANDE DOURADOS
Cálculo Diferencial e Integral II — Avaliação P2
Prof. Adriano Barbosa

Engenharia Civil 21/03/2023

Aluno(a):..

Todas as respostas devem ser justificadas.

1. Calcule a integral $\displaystyle\int_1^2 \frac{x}{x-1}\, dx$.

2. Calcule a integral definida $\displaystyle\int_0^1 x\sqrt{x^2+1}\, dx$.

3. Calcule a integral indefinida $\displaystyle\int e^x \operatorname{sen}(\pi - x)\, dx$.

4. Calcule a integral $\displaystyle\int \frac{dx}{x^2-a^2}$, onde $a \neq 0$.

5. (a) Calcule a integral $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}\, dx$.

   (b) Determine se a integral imprópria $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}\, dx$ é convergente e dê seu valor se possível.

Boa Prova!

Solução P2

① Observe que $f(x) = \frac{x}{x-1}$ não está definido em $x=1$, logo

$$\int_0^1 \frac{x}{x-1}\,dx = \lim_{t \to 1^-} \int_0^t \frac{x}{x-1}\,dx$$

onde, tomando $u = x-1$, temos $x = u+1$ e $du = dx$:

$$\int \frac{x}{x-1}\,dx = \int \frac{u+1}{u}\,du = \int 1 + \frac{1}{u}\,du = \int du + \int \frac{1}{u}\,du$$

$$= u + \ln|u| + C = x - 1 + \ln|x-1| + C$$

$$\Rightarrow \int_0^t \frac{x}{x-1}\,dx = \left(x - 1 + \ln|x-1| \Big|_0^t \right) = t - 1 + \ln|t-1| - 0 + 1 - \ln|0-1|$$

$$= t + \ln|t-1|$$

Portanto,

$$\int_0^1 \frac{x}{x-1}\,dx = \lim_{t \to 1^-} \left(t + \ln|t-1| \right) = -\infty$$

② Tome $u = x^2+1$, logo $du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}\,du$. Além disso, $x = 0 \Rightarrow u = 1$ e $x = 1 \Rightarrow u = 2$:

$$\int_0^1 x\sqrt{x^2+1}\,dx = \int_1^2 \sqrt{u} \cdot \frac{1}{2}\,du = \frac{1}{2}\int_1^2 u^{1/2}\,du = \frac{1}{2} \cdot \frac{2}{3} \left(u^{3/2} \Big|_1^2 \right)$$

$$= \frac{1}{3}\left(2^{3/2} - 1^{3/2} \right) = \frac{1}{3}\left(\sqrt{8} - 1 \right) = \frac{2\sqrt{2} - 1}{3}.$$

③ Integrando por partes:

$$u = \text{sen}(\pi - x) \qquad \Rightarrow \qquad du = -\cos(\pi-x)\,dx$$
$$dv = e^x \qquad \qquad v = e^x$$

$$\therefore \int e^x \,\text{sen}(\pi-x)\,dx = e^x\,\text{sen}(\pi-x) + \int e^x \cos(\pi-x)\,dx$$

Integrando por partes novamente:

$$u = \cos(\pi-x) \qquad \Rightarrow \qquad du = \text{sen}(\pi-x)\,dx$$
$$dv = e^x \qquad \qquad v = e^x$$

$$\therefore \int e^x\,\text{sen}(\pi-x)\,dx = e^x\,\text{sen}(\pi-x) + \left[e^x\cos(\pi-x) - \int e^x\,\text{sen}(\pi-x)\,dx \right]$$

$$\Rightarrow 2\int e^x\,\text{sen}(\pi-x)\,dx = e^x\left[\text{sen}(\pi-x)+\cos(\pi-x)\right]$$

$$\Rightarrow \int e^x\,\text{sen}(\pi-x)\,dx = \frac{e^x}{2}\left[\text{sen}(\pi-x)+\cos(\pi-x)\right] + C.$$

④ Temos que $x^2-a^2 = (x-a)(x+a)$, logo:

$$\frac{1}{x^2-a^2} = \frac{A}{x-a} + \frac{B}{x+a} = \frac{A(x+a)+B(x-a)}{(x-a)(x+a)}$$

$$\Rightarrow 1 = A(x+a) + B(x-a) = Ax + Aa + Bx - Ba = (A+B)x + (A-B)a$$

$$\Rightarrow \begin{cases} A+B=0 \Rightarrow B=-A \\ (A-B)a = 1 \end{cases} \Rightarrow (A+A)a = 1 \Rightarrow 2Aa = 1 \Rightarrow A = \frac{1}{2a}$$

$$\Rightarrow B = -\frac{1}{2a}$$

Assim,

$$\frac{1}{x^2-a^2} = \frac{1}{2a}\cdot\frac{1}{x-a} - \frac{1}{2a}\cdot\frac{1}{x+a} = \frac{1}{2a}\left(\frac{1}{x-a}-\frac{1}{x+a}\right)$$

$$\Rightarrow \int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\left(\int\frac{1}{x-a}\,dx - \int\frac{1}{x+a}\,dx\right)$$

$$= \frac{1}{2a}\left(\ln|x-a| - \ln|x+a|\right) + C.$$

5 a) Chame $u = x^3$, logo $du = 3x^2\,dx \Rightarrow x^2\,dx = \frac{1}{3}\,du$:

$$\int \frac{x^2}{9+x^6}\,dx = \int \frac{x^2}{9+(x^3)^2}\,dx = \frac{1}{3}\int\frac{1}{9+u^2}\,du = \frac{1}{3}\left[\frac{1}{3}\operatorname{arctg}\left(\frac{u}{3}\right)\right]+C$$

$$= \frac{1}{9}\operatorname{arctg}\left(\frac{x^3}{3}\right)+C.$$

b) $$I = \int_{-\infty}^{0}\frac{x^2}{9+x^6}\,dx = \lim_{t\to-\infty}\left[\frac{1}{9}\operatorname{arctg}\left(\frac{x^3}{3}\right)\Big|_t^0\right]$$

$$= \lim_{t\to-\infty}\frac{1}{9}\left[\operatorname{arctg}(0) - \operatorname{arctg}\left(\frac{t^3}{3}\right)\right] = \lim_{t\to-\infty} -\frac{1}{9}\operatorname{arctg}\left(\frac{t^3}{3}\right) = \frac{\pi}{18}$$

$$II = \int_{0}^{\infty}\frac{x^2}{9+x^6}\,dx = \lim_{t\to\infty}\left[\frac{1}{9}\operatorname{arctg}\left(\frac{x^3}{3}\right)\Big|_0^t\right]$$

$$= \lim_{t\to\infty}\frac{1}{9}\left[\operatorname{arctg}\left(\frac{t^3}{3}\right) - \operatorname{arctg}(0)\right] = \lim_{t\to\infty}\frac{1}{9}\operatorname{arctg}\left(\frac{t^3}{3}\right) = \frac{\pi}{18}$$

Portanto,

$$\int_{-\infty}^{\infty}\frac{x^2}{9+x^6}\,dx = I + II = \frac{\pi}{9}.$$