Equações separável

Uma EDO é dita separável se pode ser escrita na forma

$$\underbrace{f(y)}_{\text{não dep. de } x}\ \underbrace{\frac{dy}{dx}}_{y'} = \underbrace{g(x)}_{\text{não dep. de } y}.$$

$y(x)$, $\forall x \in I \subset \mathbb{R}$

Para resolver, fazemos

$$f(y)\frac{dy}{dx} = g(x) \Rightarrow \int f(y)\,dy = \int g(x)\,dx$$

$u = y(x) \Rightarrow du = y'\,dx$

$$f(y)\,y' = g(x) \Rightarrow \int f(y)\,y'\,dx = \int g(x)\,dx \Rightarrow \int f(u)\,du = \int g(x)\,dx$$

Exemplos: 1) $\frac{dy}{dx} = \frac{x^2}{y^2} \overset{(xy^2)}{\Rightarrow} \underbrace{y^2}_{f(y)}\frac{dy}{dx} = \underbrace{x^2}_{g(x)}$ é separável.

$$\Rightarrow \int y^2\,dy = \int x^2\,dx \Rightarrow \frac{y^3}{3} + C_1 = \frac{x^3}{3} + C_2 \Rightarrow \frac{y^3}{3} = \frac{x^3}{3} + C_2 - C_1$$

$$\Rightarrow y^3 = x^3 + C \Rightarrow y = \sqrt[3]{x^3+C} = (x^3+C)^{1/3}.$$

Verificando:

$$y' = \frac{1}{3}(x^3+C)^{-\frac{2}{3}} \cdot 3x^2 = x^2(x^3+C)^{-\frac{2}{3}} = \frac{x^2}{(x^3+C)^{2/3}} = \frac{x^2}{\left[(x^3+C)^{1/3}\right]^2} = \frac{x^2}{y^2}$$

2) $y' = \dfrac{6x^2}{2y+\cos y} \Rightarrow \underbrace{(2y+\cos y)}_{f(y)}\, y' = \underbrace{6x^2}_{g(x)}$ é separável

$\therefore \int 2y+\cos y\, dy = \int 6x^2 dx \Rightarrow y^2 + \operatorname{sen} y = 2x^3 + C.$

é sol. implícita da EDO.

Verificando:

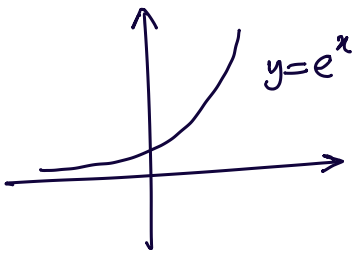
$(y^2+\operatorname{sen} y)' = (2x^3+C)' \Rightarrow 2yy' + \cos y\, y' = 6x^2 + 0$

$\Rightarrow y'(2y+\cos y) = 6x^2 \Rightarrow y' = \dfrac{6x^2}{2y+\cos y}$

3) $y' = x^2 y \overset{(\div y)}{\underset{(y\neq 0)}{\Rightarrow}} \dfrac{1}{y} y' = x^2$ é separável

$\therefore \int \dfrac{1}{y} dy = \int x^2 dx \Rightarrow \ln|y| = \dfrac{x^3}{3} + C \Rightarrow |y| = e^{\frac{x^3}{3}+C}$

$\Rightarrow e^{\frac{x^3}{3}+C} = \begin{cases} y, & y>0 \\ -y, & y<0 \end{cases} \Rightarrow \pm y = e^{\frac{x^3}{3}+C} \Rightarrow y = \pm e^{\frac{x^3}{3}+C}$



$y = 0 \Rightarrow y' = 0 \therefore y = 0$ é sol. da EDO.

Observe que:

$y = \pm e^{\frac{x^3}{3}+C} = \pm e^{\frac{x^3}{3}} \cdot \underbrace{e^C}_{>0} = k e^{\frac{x^3}{3}}, \quad k \in \mathbb{R}$

$$x\sqrt{x+1} = 0 \quad \overset{(\div x)}{\underset{(x\neq 0)}{\Longleftrightarrow}} \quad \sqrt{x+1} = 0 \quad \Longleftrightarrow \left(\sqrt{x+1}\right)^2 = 0^2$$

$$\Longleftrightarrow x+1 = 0 \Longleftrightarrow x = -1$$

$$(-1)\sqrt{(-1)+1} = (-1)\sqrt{0} = 0$$

$$\sqrt{x+1}\cdot\sqrt{x+1} = (x+1)^{1/2}\cdot(x+1)^{1/2} = (x+1)^{\frac{1}{2}+\frac{1}{2}} = (x+1)^1 = x+1$$

$$\frac{x\sqrt{x+1}}{x} = \frac{0}{x} \Longrightarrow \sqrt{x+1} = 0$$

$$x+1-1 = 0-1 \Rightarrow x = -1$$