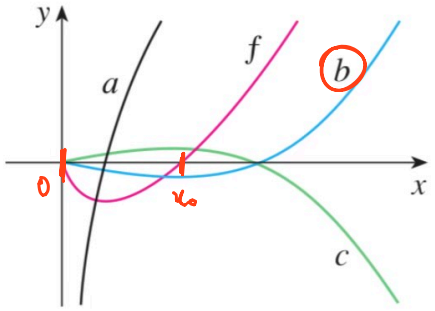
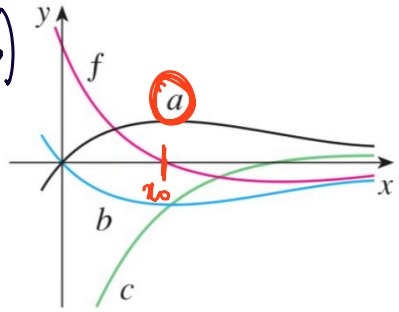
(3) O gráfico de uma função f é dado em cada item. Determine qual dos gráficos a, b ou c é a antiderivada de f.

a)



b)



$f' < 0 \Rightarrow f$ é decrescente

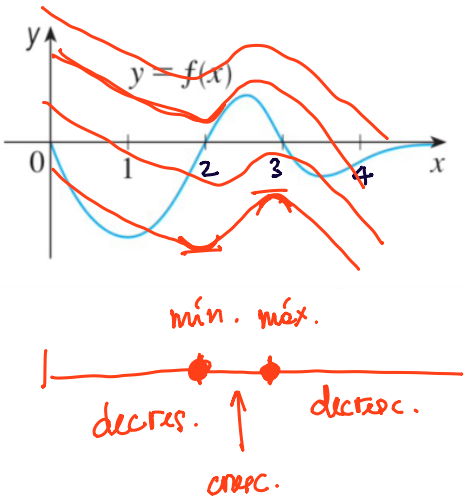
$f' > 0 \Rightarrow f$ é crescente

$f' = 0 \Rightarrow$ tangente horizontal

a) Como f é negativa em $[0, x_0]$, temos que sua antiderivada deve ser decrescente. Assim, a única função que atende esse requisito é a função b.

b) Como p/ $x < x_0$ f é positiva, sua antiderivada deve ser crescente. Analogamente, f é negativa p/ $x > x_0$, logo sua antiderivada é decresc. Portanto, sua antid. é a.

(4) Como deve ser o gráfico de uma antiderivada de f se o gráfico de f for



L3,2d) $\int \sqrt{x}\cdot \operatorname{sen}(1+x^{3/2})\,dx = \int x^{\frac{1}{2}} \operatorname{sen}(1+x^{3/2})\,dx$

$\left(\frac{3}{2}-\frac{2}{2}=\frac{1}{2}\right)$

$\left(\frac{1}{3/2}=1\cdot\frac{2}{3}\right)$

$u = 1+x^{3/2} \Rightarrow du = \frac{3}{2}x^{1/2}dx \Rightarrow x^{\frac{1}{2}}dx = \frac{1}{3/2}du = \frac{2}{3}du$

$\therefore \int x^{1/2}\operatorname{sen}(1+x^{3/2})\,dx = \int \operatorname{sen}u\cdot\frac{2}{3}du = \frac{2}{3}\int \operatorname{sen}u\,du$

$= \frac{2}{3}(-\cos u)+C = -\frac{2}{3}\cos(1+x^{3/2})+C.$

L5,1a) $\int_3^{\infty}\frac{1}{(x-2)^{3/2}}dx = \lim_{t\to\infty}\int_3^t \frac{1}{(x-2)^{3/2}}dx$

Calculando a primitiva:

$-\frac{3}{2}+\frac{2}{2}=-\frac{1}{2}$

$\int \frac{1}{(x-2)^{3/2}}dx = \int (x-2)^{-3/2}dx$

$u = x-2 \Rightarrow du = dx$

$\therefore \int (x-2)^{-3/2}dx = \int u^{-3/2}du = \frac{u^{-1/2}}{-1/2}+C = -2(x-2)^{-1/2}+C$

$= \frac{-2}{(x-2)^{1/2}}+C = \frac{-2}{\sqrt{x-2}}+C$

$(x>2)$

Assim,

$\int_3^t \frac{1}{(x-2)^{3/2}}dx = -2(x-2)^{-1/2}\Big|_3^t = -2(t-2)^{-1/2}+2(3-2)^{-1/2}$

$= -2(t-2)^{-1/2}+2$

Portanto,

$\int_3^{\infty}\frac{1}{(x-2)^{3/2}}dx = \lim_{t\to\infty}\int_3^t\frac{1}{(x-2)^{3/2}}dx = \lim_{t\to\infty}\left[-2(t-2)^{-1/2}+2\right] = 2.$

L4, 1b) $\int \theta \cos\theta \, d\theta = \int x \cos x \, dx = \int t \cos t \, dt$

Integrando por partes:

$u = \theta \quad \Rightarrow \quad du = d\theta$

$dv = \cos\theta \, d\theta \qquad v = \operatorname{sen}\theta$

$\therefore \int \theta \cos\theta \, d\theta = \theta \operatorname{sen}\theta - \int \operatorname{sen}\theta \, d\theta = \theta \operatorname{sen}\theta + \cos\theta + C.$

L1, 2) $f''(x) = 2 + \cos x \,, \quad f(0) = -1 \,, \quad f\left(\frac{\pi}{2}\right) = 0$

$f'(x) = 2x + \operatorname{sen} x + C$

$f(x) = x^2 - \cos x + Cx + K$

$$\begin{cases} -1 = f(0) = 0^2 - \cos 0 + C \cdot 0 + K \\ 0 = f\left(\frac{\pi}{2}\right) = \left(\frac{\pi}{2}\right)^2 - \cos\left(\frac{\pi}{2}\right) + C \cdot \frac{\pi}{2} + K \end{cases} \Rightarrow \begin{cases} -1 + K = -1 \Rightarrow \boxed{K = 0} \\ \frac{\pi^2}{4} + \frac{\pi}{2} C + K = 0 \end{cases}$$

$\Rightarrow \frac{\pi^2}{4} + \frac{\pi}{2} C = 0 \Rightarrow \frac{\pi}{2} C = -\frac{\pi^2}{4} \Rightarrow C = -\frac{\pi^2}{4} \cdot \frac{2}{\pi} \Rightarrow \boxed{C = -\frac{\pi}{2}}$

$\therefore f(x) = x^2 - \cos x - \frac{\pi}{2} x$

L3, 2c) $\int \sec^2\theta \, tg^3\theta \, d\theta$ $\qquad (tg\theta)' = \left(\frac{\operatorname{sen}\theta}{\cos\theta}\right)' = \ldots = \sec^2\theta$

$u = tg\theta \Rightarrow du = \sec^2\theta \, d\theta$

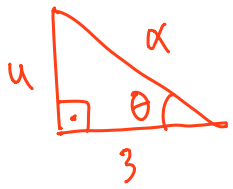
$$\therefore \int \sec^2\theta \cdot tg^3\theta \, d\theta = \int u^3 du \Rightarrow \frac{u^4}{4} + c = \frac{1}{4} tg^4\theta + c$$

L5, 1e) $\int_{-\infty}^{\infty} \frac{x^2}{9+x^6} dx$

$$\int \frac{x^2}{9+x^6} dx = \int \frac{x^2}{9+(x^3)^2} dx \qquad u = x^3$$

$$du = 3x^2 dx \Rightarrow x^2 dx = \frac{1}{3} du$$

$$\therefore \int \frac{x^2}{9+(x^3)^2} dx = \int \frac{1}{3} \cdot \frac{1}{9+u^2} du = \frac{1}{3} \int \frac{1}{9+u^2} du = \frac{1}{9} \operatorname{arctg}\left(\frac{u}{3}\right) + c$$



$$\alpha^2 = u^2 + 9$$

$$tg\theta = \frac{u}{3} \Rightarrow u = 3tg\theta$$