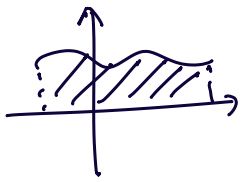
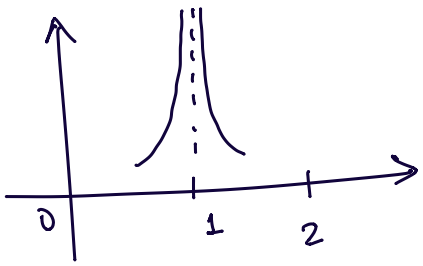
$$\int_0^2 \frac{1}{(x-1)^2}\,dx \qquad (x-1)^2 \geq 0 \overset{(x\neq 1)}{\Rightarrow} \frac{1}{(x-1)^2} > 0$$



$u = x-1 \Rightarrow du = dx$ e $x=0 \Rightarrow u=-1$, $x=2 \Rightarrow u=1$

$$\therefore \int_0^2 \frac{1}{(x-1)^2}\,dx = \int_{-1}^{1} \frac{1}{u^2}\,du = \int_{-1}^{1} u^{-2}\,du = \frac{u^{-1}}{-1}\Big|_{-1}^{1} = -\frac{1}{u}\Big|_{-1}^{1}$$

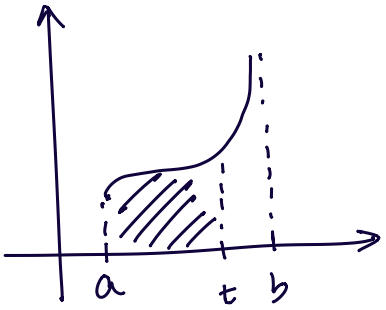
$$= -\frac{1}{1} + \frac{1}{(-1)} = -1-1 = -2 < 0$$



$x \to 1^- \Rightarrow x-1 \to 0^- \Rightarrow (x-1)^2 \to 0^+ \Rightarrow \frac{1}{(x-1)^2} \to +\infty$

$x \to 1^+ \Rightarrow x-1 \to 0^+ \Rightarrow (x-1)^2 \to 0^+ \Rightarrow \frac{1}{(x-2)^2} \to +\infty$

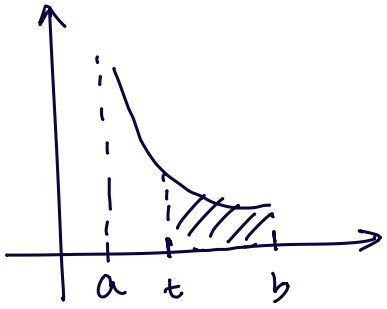
A integral é imprópria!



$f: [a,b) \to \mathbb{R}$ contínua com assíntota vertical em b

Se $\int_a^t f(x)\,dx$, $\forall\, t<b$, existe

$$\int_a^b f(x)\,dx = \lim_{t\to b^-} \int_a^t f(x)\,dx,$$ desde que o limite exista.



$f: (a,b] \to \mathbb{R}$ contínua com assíntota vertical em a

Se $\int_t^b f(x)\,dx$, $\forall\, t>a$, existe

$$\int_a^b f(x)\,dx = \lim_{t\to a^+} \int_t^b f(x)\,dx,$$ desde que o limite exista.

Exemplos: 1) $\int_2^5 \frac{1}{\sqrt{x-2}}\,dx$ $\qquad x-2 \geq 0 \Leftrightarrow x \geq 2$ e $x \neq 2$

$f(x) = \frac{1}{\sqrt{x-2}}$ está bem def. p/ $x > 2$. Além disso, f é contínua para $x > 2$, pois é quociente de contínuas (composta)

$$\int_2^5 \frac{1}{\sqrt{x-2}}\,dx \overset{(def.)}{=} \lim_{t \to 2^+} \int_t^5 \frac{1}{\sqrt{x-2}}\,dx$$

$$\int \frac{1}{\sqrt{x-2}}\,dx \qquad u = x-2 \Rightarrow du = dx$$

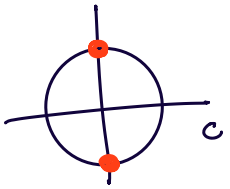
$$= \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} + C = 2\sqrt{x-2} + C$$

$$\Rightarrow \int_t^5 \frac{1}{\sqrt{x-2}}\,dx = 2\sqrt{x-2}\Big|_t^5 = 2\sqrt{5-2} - 2\sqrt{t-2} = 2\sqrt{3} - 2\sqrt{t-2}$$

Assim,

$$\int_2^5 \frac{1}{\sqrt{x-2}}\,dx = \lim_{t \to 2^+} \left(2\sqrt{3} - 2\sqrt{t-2}\right) = 2\sqrt{3}.$$

2) $\int_0^{\frac{\pi}{2}} \sec x\,dx$ $\qquad \sec x = \frac{1}{\cos x}$



$x = (2k+1)\frac{\pi}{2}$

$$\overset{(def.)}{=} \lim_{t \to \frac{\pi}{2}^-} \int_0^t \sec x\,dx$$

$$\int \sec x\,dx = \int \sec x \cdot \frac{\sec x + \operatorname{tg} x}{\sec x + \operatorname{tg} x}\,dx = \int \frac{\sec^2 x + \sec x \cdot \operatorname{tg} x}{\sec x + \operatorname{tg} x}\,dx$$

$$u = \sec x + \operatorname{tg} x \quad \Rightarrow du = (\sec x \cdot \operatorname{tg} x + \sec^2 x)\,dx$$

$$\therefore \int \sec x\,dx = \int \frac{\sec^2 x + \sec x \cdot \operatorname{tg} x}{\sec x + \operatorname{tg} x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C$$

$$= \ln|\sec x + \operatorname{tg} x| + C.$$

$$\Rightarrow \int_0^t \sec x\,dx = \ln|\sec x + \operatorname{tg} x|\Big|_0^t$$

$$= \ln|\sec t + \operatorname{tg} t| - \ln|\sec 0 + \operatorname{tg} 0|$$

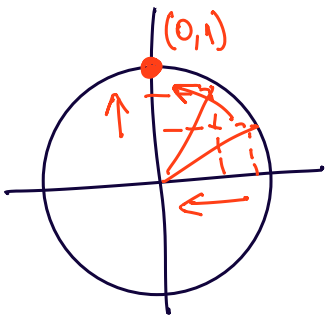
($\sec 0 = 1$, $\operatorname{tg} 0 = 0$)

Assim,

$$\int_0^{\pi/2} \sec x\,dx = \lim_{t \to \frac{\pi}{2}^-} \left(\ln|\sec t + \operatorname{tg} t|\right)$$

$$= +\infty$$

Portanto, a integral diverge.



$t \to \frac{\pi}{2}^-$

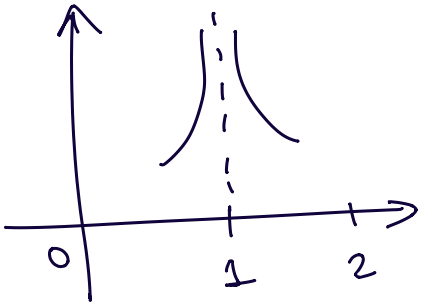
$\operatorname{sen} t \to 1^-$

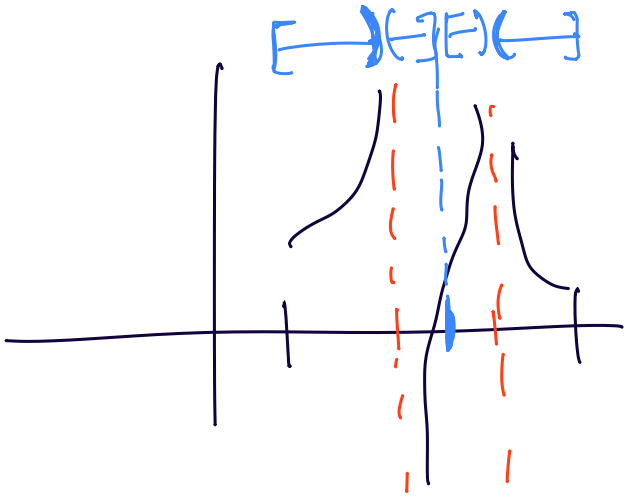
$\cos t \to 0^+$

$\frac{1}{\cos t} = \sec t \to +\infty$

$\frac{\operatorname{sen} t}{\cos t} = \operatorname{tg} t \to +\infty$

$$\int_0^2 \frac{1}{(x-1)^2}\,dx$$



$$= \int_0^{\underline{1}} \frac{1}{(x-1)^2}\,dx + \int_{\underline{1}}^2 \frac{1}{(x-1)^2}\,dx$$

exercício!