1) $\int_1^2 x^4 (\ln x)^2 \, dx$

Integrando por partes:

$u = (\ln x)^2 \quad \Rightarrow \quad du = 2\ln x \cdot \frac{1}{x} dx$

$dv = x^4 dx \quad \Rightarrow \quad v = \frac{x^5}{5}$

$u = x^4 \quad \Rightarrow \quad du = 4x^3 dx$

$dv = (\ln x)^2 dx \quad \Rightarrow \quad v = \int u^2 e^u du$

$\left(\begin{array}{l} u = \ln x \Rightarrow du = \frac{1}{x} dx \\ x = e^u \Rightarrow dx = e^u du \end{array} \right)$

$$\therefore \int x^4 (\ln x)^2 dx = \frac{x^5 (\ln x)^2}{5} - \int \frac{x^5}{5} \cdot \frac{1}{x} \cdot 2\ln x \, dx$$

$$= \frac{x^5 (\ln x)^2}{5} - \frac{2}{5} \int x^4 \ln x \, dx$$

Integrando por partes novamente:

$u = \ln x \quad \Rightarrow \quad du = \frac{1}{x} dx$

$dv = x^4 dx \quad \Rightarrow \quad v = \frac{x^5}{5}$

$$\therefore \int x^4 (\ln x)^2 dx = \frac{x^5 (\ln x)^2}{5} - \frac{2}{5} \left(\frac{x^5 \ln x}{5} - \int \frac{x^5}{5} \cdot \frac{1}{x} dx \right)$$

$$= \frac{x^5 (\ln x)^2}{5} - \frac{2x^5 \ln x}{25} + \frac{2}{25} \int x^4 dx$$

$$= \frac{x^5 (\ln x)^2}{5} - \frac{2x^5 \ln x}{25} + \frac{2x^5}{125} + C$$

Assim,

$$\int_1^2 x^4 (\ln x)^2 \, dx = \frac{2^5 (\ln 2)^2}{5} - \frac{2 \cdot 2^5 \ln 2}{25} + \frac{2 \cdot 2^5}{125} - \frac{1^5 (\ln 1)^2}{5} + \frac{2 \cdot 1^5 \ln 1}{25} - \frac{2 \cdot 1^5}{125}$$

$$= \frac{32}{5}(\ln 2)^2 - \frac{64}{25}\ln 2 + \frac{64}{125} - \frac{2}{125}$$

$$= \frac{32}{5}(\ln 2)^2 - \frac{64}{25}\ln 2 + \frac{62}{125}$$

$$\int_1^2 x^4 \log^2(x)\, dx = \frac{2}{125}\left(31 + 400\log^2(2) - 160\log(2)\right) \approx 1.7964$$

2) $\int x^3 e^{x^2} dx$

$u = x^3$

$dv = e^{x^2} dx \Rightarrow v = \int e^{x^2} dx$???

$$\int x^3 e^{x^2} dx = \int x^2 \cdot x e^{x^2} dx$$

$u = x^2$ $\quad\Rightarrow\quad$ $du = 2x\,dx$

$dv = x e^{x^2} dx$ $\quad\Rightarrow\quad$ $v = \int x e^{x^2} dx$

$= \int e^u \cdot \frac{1}{2} du = \frac{1}{2} e^{x^2}$

$\left(\begin{array}{l} u = x^2 \\ du = 2x\,dx \end{array}\right)$

$$\therefore \int x^3 e^{x^2} dx = \frac{1}{2} x^2 e^{x^2} - \int \frac{1}{2} e^{x^2} \cdot 2x\,dx$$

$$= \frac{1}{2} x^2 e^{x^2} - \int x e^{x^2} dx$$

$$= \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + C$$