1) $\int e^x \operatorname{sen} x \, dx = F(x)$ (função)

Integrando por partes:

$u = e^x \qquad du = e^x dx$

$\Rightarrow$

$dv = \operatorname{sen} x \, dx \qquad v = -\cos x$

$\therefore \int e^x \operatorname{sen} x \, dx = -e^x \cos x + \int e^x \cos x \, dx$

Integrando por partes novamente:

$u = e^x \qquad du = e^x dx$

$\Rightarrow$

$dv = \cos x \, dx \qquad v = \operatorname{sen} x$

$\therefore \int e^x \operatorname{sen} x \, dx = -e^x \cos x + e^x \operatorname{sen} x - \int e^x \operatorname{sen} x \, dx$

$F(x) = -e^x \cos x + e^x \operatorname{sen} x - F(x) \Rightarrow 2F(x) = -e^x \cos x + e^x \operatorname{sen} x$

$\Rightarrow F(x) = \dfrac{-e^x \cos x + e^x \operatorname{sen} x}{2}$

Portanto, $\int e^x \operatorname{sen} x \, dx = \dfrac{-e^x \cos x + e^x \operatorname{sen} x}{2} + C$

Verificando:

$F'(x) = \frac{1}{2}\left(-\cancel{e^x \cos x} + e^x \operatorname{sen} x + e^x \operatorname{sen} x + \cancel{e^x \cos x}\right)$

$= e^x \operatorname{sen} x.$

Sol. 2:

Integrando por partes:

$$u = \operatorname{sen} x \qquad du = \cos x\, dx$$
$$dv = e^x dx \Rightarrow \qquad v = e^x$$

$$\therefore \int e^x \operatorname{sen} x\, dx = e^x \operatorname{sen} x - \int e^x \cos x\, dx$$

Integrando por partes novamente:

$$u = \cos x \qquad du = -\operatorname{sen} x\, dx$$
$$dv = e^x dx \Rightarrow \qquad v = e^x$$

$$\therefore \int e^x \operatorname{sen} x\, dx = e^x \operatorname{sen} x - \left(e^x \cos x + \int e^x \operatorname{sen} x\, dx \right)$$
$$= e^x \operatorname{sen} x - e^x \cos x - \int e^x \operatorname{sen} x\, dx$$

$$\Rightarrow 2 \int e^x \operatorname{sen} x\, dx = e^x \operatorname{sen} x - e^x \cos x$$

$$\Rightarrow \int e^x \operatorname{sen} x\, dx = \frac{e^x \operatorname{sen} x - e^x \cos x}{2} + C$$

Sol. 3: Integrando por partes:

$$u = e^x \qquad du = e^x dx$$

$$\Rightarrow$$

$$dv = \operatorname{sen} x\, dx \qquad v = -\cos x$$

$$\therefore \int e^x \operatorname{sen} x\, dx = -e^x \cos x + \int e^x \cos x\, dx$$

Integrando por partes novamente:

$$u = \cos x \qquad du = -\operatorname{sen} x\, dx$$

$$\Rightarrow$$

$$dv = e^x dx \qquad v = e^x$$

$$\therefore \int e^x \operatorname{sen} x\, dx = -e^x \cos x + e^x \cos x + \int e^x \operatorname{sen} x\, dx$$

$$\Rightarrow \quad 0 = 0 \quad \checkmark$$

$$\int u\,dv = uv - \int v\,du$$

$$\int u\,dv$$

2) $\int_0^1 \frac{y}{e^{2y}}\,dy$

Integrando por partes:

$u = y$ $\qquad du = dx$

~~$dv = e^{2y}dy$~~ $\Rightarrow$

$dv = \frac{1}{e^{2y}}dy = e^{-2y}dy$ $\qquad v = \int e^{-2y}dy$ $\quad (u = -2y,\ du = -2dy)$

$= -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u = -\frac{1}{2}e^{-2y}$

$\therefore \int_0^1 \frac{y}{e^{2y}}dy = y\cdot\left(-\frac{1}{2}e^{-2y}\right)\Big|_0^1 - \int_0^1 -\frac{1}{2}e^{-2y}dy$

$= -\frac{1}{2}\cdot\left(\frac{y}{e^{2y}}\Big|_0^1\right) + \frac{1}{2}\int_0^1 e^{-2y}dy$

$= -\frac{1}{2}\cdot\left(\frac{1}{e^2} - \frac{0}{e^0}\right) + \frac{1}{2}\cdot\left(-\frac{1}{2}\right)\cdot\left(e^{-2y}\Big|_0^1\right)$

$= -\frac{1}{2}e^{-2} - \frac{1}{4}\left(e^{-2} - e^0\right) = -\frac{1}{2}e^{-2} - \frac{1}{4}e^{-2} + \frac{1}{4}$

$= e^{-2}\left(-\frac{1}{2} - \frac{1}{4}\right) + \frac{1}{4} = -\frac{3}{4}e^{-2} + \frac{1}{4} = \frac{-3e^{-2}+1}{4}$

$$\int_0^1 \frac{y}{e^{2y}}\,dy$$

Tomando $u = 2y$, temos $du = 2\,dy \Rightarrow dy = \frac{1}{2}\,du$

e $y = \frac{u}{2}$. Além disso, $y = 0 \Rightarrow u = 0$ e $y = 1 \Rightarrow u = 2$.

$$\therefore \int_0^1 \frac{y}{e^{2y}}\,dy = \int_0^2 \frac{u/2}{e^u} \cdot \frac{1}{2}\,du = \frac{1}{4}\int_0^2 \frac{u}{e^u}\,du = \frac{1}{4}\int_0^2 u \cdot e^{-u}\,du$$

(por partes)
exercício

$$\int \frac{y}{e^{2y}}\,dy = \int y \cdot e^{-2y}\,dy$$

$u = -2y \Rightarrow du = -2\,dy \Rightarrow dy = -\frac{1}{2}\,du$

e $y = -\frac{u}{2}$

$$\therefore \int y\,e^{-2y}\,dy = \int -\frac{u}{2} \cdot e^u \cdot \left(-\frac{1}{2}\right) du = \frac{1}{4}\int u\,e^u\,du$$

(por partes)
exercício

3) $$\int_1^2 x^4 (\ln x)^2\,dx$$