1) $\int_0^4 \sqrt{2x+1}\,dx$

$u = 2x+1 \Rightarrow du = 2\,dx \Rightarrow dx = \frac{1}{2}\,du$

$x = 0 \Rightarrow u = 1$ e $x = 4 \Rightarrow u = 9$

$\therefore \int_0^4 \sqrt{2x+1}\,dx = \int_1^9 \sqrt{u}\cdot\frac{1}{2}\,du = \frac{1}{2}\int_1^9 u^{1/2}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_1^9$

$= \frac{u^{3/2}}{3}\Big|_1^9 = \frac{9^{3/2}}{3} - \frac{1^{3/2}}{3} = \frac{27}{3} - \frac{1}{3} = \frac{26}{3}$

2) $\int_1^2 \frac{dx}{(3-5x)^2}$

$u = 3-5x \Rightarrow du = -5\,dx \Rightarrow dx = -\frac{1}{5}\,du$

$x = 1 \Rightarrow u = -2$ e $x = 2 \Rightarrow u = -7$

$\therefore \int_1^2 \frac{dx}{(3-5x)^2} = \int_{-2}^{-7} \frac{1}{u^2}\cdot\left(-\frac{1}{5}\right)du = \frac{1}{5}\int_{-7}^{-2} u^{-2}\,du$

$= \frac{1}{5}\left(-u^{-1}\Big|_{-7}^{-2}\right) = \frac{1}{5}\left(-\frac{1}{u}\Big|_{-7}^{-2}\right) = \frac{1}{5}\left[-\frac{1}{(-2)} + \frac{1}{(-7)}\right]$

$= \frac{1}{5}\left[\frac{1}{2} - \frac{1}{7}\right] = \frac{1}{5}\left[\frac{7-2}{14}\right] = \frac{1}{14}.$

3) Se f é contínua e $\int_0^4 f(x)\,dx = 10$, calcule

$\int_0^2 f(2x)\,dx$.

$u = 2x \Rightarrow du = 2\,dx \Rightarrow dx = \frac{1}{2}\,du$.

$x = 0 \Rightarrow u = 0$ e $x = 2 \Rightarrow u = 4$.

$$\therefore \int_0^2 f(2x)\,dx = \int_0^4 f(u)\cdot\frac{1}{2}\,du = \frac{1}{2}\int_0^4 f(u)\,du = \frac{1}{2}\cdot 10 = 5.$$

Relembrando:

F é primitiva de f se $F'(x) = f(x)$.

$f(x) = x^2$, $F(x) = \frac{x^3}{3} + C \Rightarrow F'(x) = \frac{3x^2}{3} + 0 = x^2$.

$\int_a^b f(x)\,dx = F(b) - F(a)$, onde $F' = f$.

Integração por partes

$$[f(x)\cdot g(x)]' = f'(x)\cdot g(x) + f(x)\cdot g'(x) \quad \text{(regra do produto)}$$

$$\Rightarrow \int [f(x)\cdot g(x)]'\,dx = \int f'(x)\cdot g(x) + f(x)\cdot g'(x)\,dx$$

$$\Rightarrow \quad f(x)\cdot g(x) = \int f'(x)\cdot g(x)\,dx + \int f(x)\cdot g'(x)\,dx$$

$$\Rightarrow \int \underbrace{f(x)}_{u}\cdot \underbrace{g'(x)\,dx}_{dv} = \underbrace{f(x)}_{u}\cdot \underbrace{g(x)}_{v} - \int \underbrace{g(x)}_{v}\cdot \underbrace{f'(x)\,dx}_{du}$$

$u = f(x) \quad \Rightarrow \quad du = f'(x)\,dx$

$dv = g'(x)\,dx \quad \Leftarrow \quad v = g(x)$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Exemplos: 1) $\int \underbrace{x}_{u}\cdot \underbrace{\operatorname{sen} x\,dx}_{dv}$

$$\begin{aligned} u &= x \\ dv &= \operatorname{sen} x\,dx \end{aligned} \quad \Rightarrow \quad \begin{aligned} du &= dx \\ v &= -\cos x \end{aligned}$$

$$\therefore \int x\cdot \operatorname{sen} x\,dx = \underbrace{-x\cdot\cos x}_{uv} - \int \underbrace{-\cos x\,dx}_{v\,du}$$

$$= -x\cdot\cos x + \int \cos x\,dx$$

$$= -x\cdot\cos x + \operatorname{sen} x + C$$

Verificando:

$$(-x\cdot\cos x + \operatorname{sen} x + C)' = (-1)\cdot\cos x + (-x)\cdot(-\operatorname{sen} x) + \cos x + 0$$

$$= -\cancel{\cos x} + x\operatorname{sen} x + \cancel{\cos x} = x\operatorname{sen} x.$$

$$\int x \operatorname{sen} x \, dx$$

$$\begin{aligned} u &= \operatorname{sen} x & du &= \cos x \, dx \\ dv &= x \, dx & \Rightarrow \quad v &= \frac{x^2}{2} \end{aligned}$$

$$\therefore \int x \cdot \operatorname{sen} x \, dx = \frac{x^2 \operatorname{sen} x}{2} - \int \frac{x^2}{2} \cos x \, dx$$

$$= \frac{x^2 \operatorname{sen} x}{2} - \frac{1}{2} \int x^2 \cos x \, dx$$

du ruim!! :(

Observação: Escolhemos para u uma função que fique "mais simples" enquanto que dv seja uma função que sabemos integrar.

2) $\int \ln x \, dx$

Integrando por partes:

$$\begin{aligned} u &= \ln x & du &= \frac{1}{x} dx \\ dv &= dx & \Rightarrow \quad v &= x \end{aligned}$$

$$\therefore \int \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} dx$$

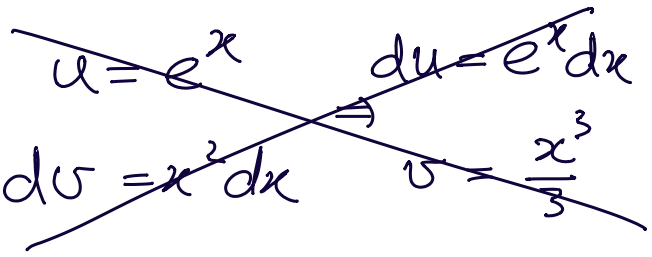
$$= x \ln x - \int 1 \, dx$$

$$= x \ln x - x + C.$$

3) $\int x^2 e^x dx$

$u = x^2 \quad \Rightarrow \quad du = 2x\,dx$

$dv = e^x dx \quad \Rightarrow \quad v = e^x$



$$\therefore \int x^2 e^x dx = x^2 e^x - \int 2x e^x dx = x^2 e^x - 2\int x e^x dx$$

Integrando por partes novamente:

$u = x \quad \Rightarrow \quad du = dx$

$dv = e^x dx \quad \Rightarrow \quad v = e^x dx$

$$\therefore \int x^2 e^x dx = x^2 e^x - 2\left(x e^x - \int e^x dx\right)$$

$$= x^2 e^x - 2x e^x - 2\int e^x dx$$

$$= x^2 e^x - 2x e^x - 2 e^x + C$$

$$= e^x (x^2 - 2x - 2) + C$$