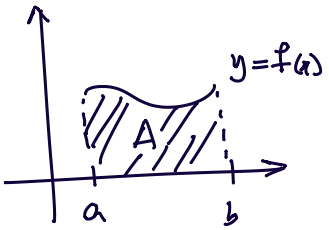
$F' = f$, F é primitiva de f. $\longrightarrow$ $F(x) = \int f(x)\,dx$ (integral indefinida)



$A = \int_a^b f(x)\,dx$ (integral definida)

área quando $f \geq 0$

"área com sinal" no caso geral

TFC: $\int_a^b f(x)\,dx = F(b) - F(a)$, onde $F' = f$.

1) $\int \frac{x}{\sqrt{1-4x^2}}\,dx$

$u = 1-4x^2 \Rightarrow du = -8x\,dx \Rightarrow x\,dx = -\frac{1}{8}\,du$

$\therefore \int \frac{x}{\sqrt{1-4x^2}}\,dx = \int \frac{1}{\sqrt{u}} \cdot \left(-\frac{1}{8}\right) du = -\frac{1}{8}\int \frac{1}{\sqrt{u}}\,du = -\frac{1}{8}\int u^{-1/2}\,du$

$= -\frac{1}{8}.2\,u^{1/2} + C = -\frac{1}{4}(1-4x^2)^{1/2} + C = -\frac{1}{4}\sqrt{1-4x^2} + C.$

2) $\int \sqrt{1+x^2}\cdot x^5\,dx = \int \sqrt{1+x^2}\cdot x^4 \cdot x\,dx$

$u = 1+x^2 \Rightarrow du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}\,du$

e $x^2 = u-1 \Rightarrow x^4 = (u-1)^2$

$\therefore \int \sqrt{1+x^2}\cdot x^5\,dx = \int \sqrt{u}\cdot(u-1)^2 \cdot \frac{1}{2}\,du = \frac{1}{2}\int u^{1/2}(u^2 - 2u + 1)\,du$

$= \frac{1}{2}\int u^{5/2} - 2u^{3/2} + u^{1/2}\,du = \frac{1}{2}\left(\frac{2}{7}u^{7/2} - 2\cdot\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C\right)$

$$= \frac{1}{7}(1+x^2)^{7/2} - \frac{2}{5}(1+x^2)^{5/2} + \frac{1}{3}(1+x^2)^{3/2} + k$$

3) $\int x(2x+5)^8 dx$

$u = 2x+5 \Rightarrow du = 2dx \Rightarrow dx = \frac{1}{2}du$

e $x = \frac{u-5}{2}$

$$\therefore \int x(2x+5)^8 dx = \int \frac{u-5}{2} \cdot u^8 \cdot \frac{1}{2} du = \frac{1}{4}\int (u-5)\cdot u^8 du$$

$$= \frac{1}{4}\int u^9 - 5u^8 du = \frac{1}{4}\left(\frac{u^{10}}{10} + 5\frac{u^9}{9} + C\right) = \frac{(2x+5)^{10}}{40} + \frac{5(2x+5)^9}{36} + k$$

4) $\int (x^2+1)(x^3+3x)^4 dx$

$u = x^3+3x \Rightarrow du = 3x^2+3\,dx = 3(x^2+1)dx$

$\Rightarrow (x^2+1)dx = \frac{1}{3}du$

$$\therefore \int (x^2+1)(x^3+3x)^4 dx = \int u^4 \cdot \frac{1}{3} du = \frac{1}{3}\int u^4 du = \frac{1}{3}\cdot\frac{u^5}{5} + C$$

$$= \frac{1}{15}(x^3+3x)^5 + C.$$

5) $\int e^{tgx} \cdot \sec^2 x \, dx$

$u = tgx \Rightarrow du = \sec^2 x \, dx$

$\therefore \int e^{tgx} \cdot \sec^2 x \, dx = \int e^u du = e^u + c = e^{tgx} + c.$

6) $\int_0^4 \sqrt{2x+1} \, dx = \frac{1}{2}\int_1^9 \sqrt{u} \, du$ exercício!

$u = 2x+1 \Rightarrow du = 2dx \Rightarrow dx = \frac{1}{2} du$

$x = 0 \Rightarrow u = 1$ e $x = 4 \Rightarrow u = 9$

$\therefore \int \sqrt{2x+1} \, dx = \int \sqrt{u} \cdot \frac{1}{2} du = \frac{1}{2}\int u^{1/2} du = \frac{1}{2}\cdot\frac{2}{3} u^{3/2} + c$

$= \frac{1}{3}(2x+1)^{3/2} + c = F(x)$

TFC: $\int_0^4 \sqrt{2x+1} \, dx = F(4) - F(0)$

$= \frac{1}{3}(2\cdot 4+1)^{3/2} + \cancel{c} - \frac{1}{3}(2\cdot 0+1)^{3/2} - \cancel{c}$

$= \frac{1}{3}\cdot 9^{3/2} - \frac{1}{3}\cdot 1^{3/2} = \frac{1}{3}\cdot 27 - \frac{1}{3}\cdot 1 = \frac{26}{3}$