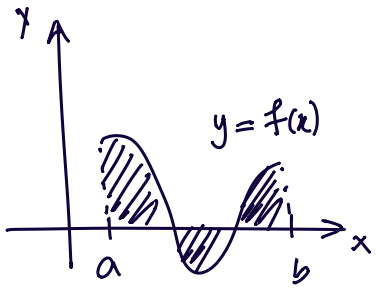
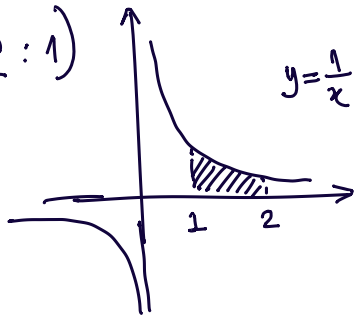


$$\int_a^b f(x)\,dx = F(b) - F(a)$$

onde, $F'(x) = f(x)$.

Exemplo: 1)



$$A = \int_1^2 \frac{1}{x}\,dx = \ln x \Big|_1^2 = \ln 2 - \cancel{\ln 1}^{\,0}$$

$$\therefore A = \ln 2 \approx 0{,}6931.$$

2) $\int e^{x^2} dx = ?$ exercício!!

Técnicas de integração: substituição

$$[F(g(x))]' = F'(g(x)) \cdot g'(x) \quad \text{(regra da cadeia)}$$

Se F é primitiva de f:

$$\int f(g(x)) \cdot g'(x)\,dx = \int F'(g(x)) \cdot g'(x)\,dx = \int [F(g(x))]'\,dx$$

$$= F(g(x)) + C$$

Assim, tomando $u = g(x)$, temos $du = g'(x)\,dx$:

$$\int f(g(x)) \cdot g'(x)\,dx = \int f(u)\,du = F(u) + C.$$

Exemplos: 1) $\int 2x\sqrt{1+x^2}\,dx$

$f(x)=\sqrt{x} = x^{1/2} \Rightarrow F(x) = \frac{2x^{3/2}}{3} + C$

$g(x) = 1+x^2 \Rightarrow g'(x) = 2x$

$\therefore \int \sqrt{1+x^2}\cdot 2x\,dx = \int f(g(x))\cdot g'(x)\,dx$

Tome $u = 1+x^2$, logo $du = 2x\,dx$:

$$\int \sqrt{1+x^2}\cdot 2x\,dx = \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(1+x^2)^{3/2} + C.$$

Verificando:

$$\left[\frac{2}{3}(1+x^2)^{3/2} + C\right]' = \frac{2}{3}\cdot\frac{3}{2}(1+x^2)^{\frac{1}{2}}\cdot 2x + 0 = \sqrt{1+x^2}\cdot 2x$$

2) $\int x^2\cdot\cos(x^3)\,dx$

Tome $u = x^3$, logo $du = 3x^2dx \Rightarrow x^2dx = \frac{1}{3}du$

$$\int \cos u\cdot\frac{1}{3}\,du = \frac{1}{3}\int \cos u\,du = \frac{1}{3}\operatorname{sen} u + C = \frac{1}{3}\operatorname{sen}(x^3) + C.$$

3) $\int \sqrt{2x+1}\,dx$

Solução 1: $u = 2x+1 \Rightarrow du = 2\,dx \Rightarrow dx = \frac{1}{2}du$

$$\int \sqrt{2x+1}\,dx = \int \sqrt{u}\cdot\frac{1}{2}\,du = \frac{1}{2}\int u^{1/2}\,du$$

$$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = \frac{1}{3}(2x+1)^{3/2} + C.$$

Solução 2: $u = \sqrt{2x+1} \Rightarrow du = \dfrac{1}{2\sqrt{2x+1}}\cdot 2\,dx = \dfrac{1}{u}\,dx$

$$\Rightarrow dx = u\,du$$

$$\therefore \int \sqrt{2x+1}\,dx = \int u\cdot u\,du = \int u^2\,du = \frac{u^3}{3} + C$$

$$= \frac{(\sqrt{2x+1})^3}{3} + C$$

4) $\int \cos(5x)\,dx$

$u = 5x \Rightarrow du = 5\,dx \Rightarrow dx = \dfrac{1}{5}\,du$

$$\therefore \int \cos(5x)\,dx = \int \cos u\cdot\frac{1}{5}\,du = \frac{1}{5}\int \cos u\,du$$

$$= \frac{1}{5}\operatorname{sen} u + C = \frac{1}{5}\operatorname{sen}(5x) + C$$

5) $\int \dfrac{x}{\sqrt{1-4x^2}}\,dx$