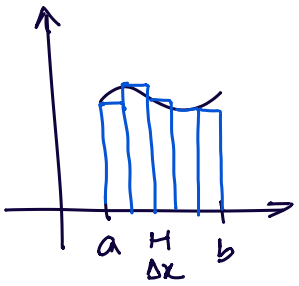
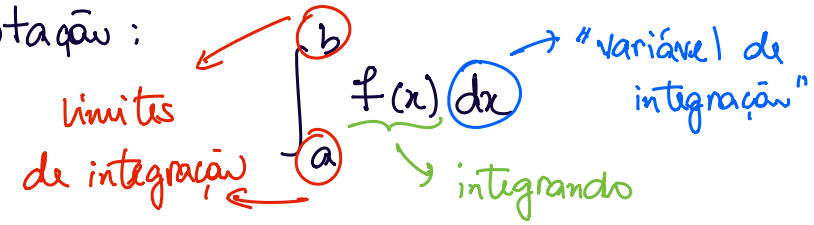


$$\Delta x = \frac{b-a}{n}$$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{f(x_i^*)\,\Delta x}_{A_i}$$

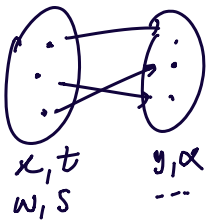
integral definida

Notação:

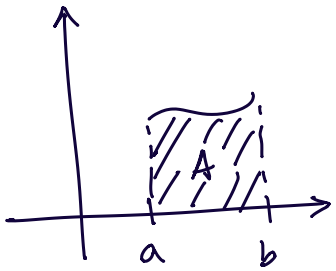


$$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt = \int_a^b f(y)\,dy = \int_a^b f(\star)\,d\star \in \mathbb{R}$$

$f: X \longrightarrow Y$

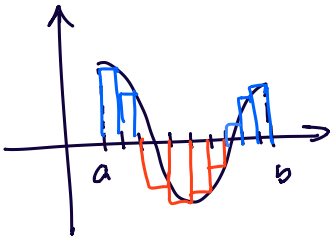


Se $f(x) \geq 0$ em $[a,b]$:



$$\int_a^b f(x)\,dx = A$$

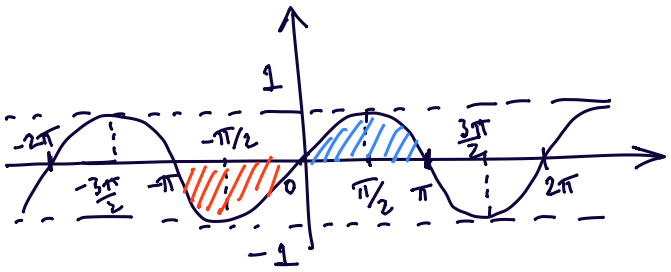
Se f tem valores negativos em $[a,b]$, ao calcular a área entre o gráfico de f, o eixo x e as retas $x=a$ e $x=b$:

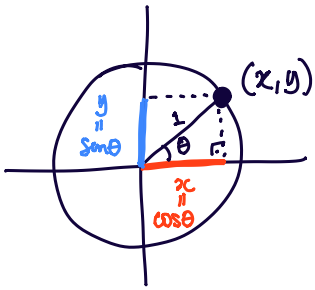


$$A = \lim_{n\to\infty} \sum_{i=1}^{n} A_i \,, \qquad A_i = \underbrace{f(x_i)}_{\substack{>0 \\ <0}} \underbrace{\Delta x}_{>0} \quad \begin{matrix} >0 \\ <0 \end{matrix}$$

$$\int_a^b f(x)\,dx = A - A \quad \text{(área com sinal)}$$

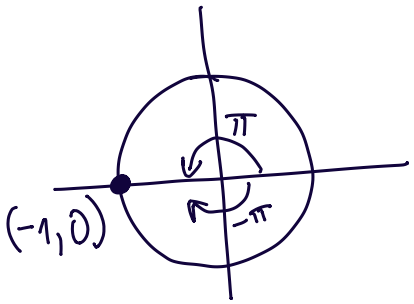
Exemplo: $\displaystyle\int_{-\pi}^{\pi} \operatorname{sen} x\,dx = A - A = 0$





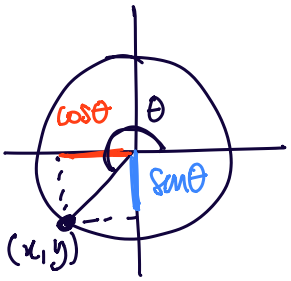
$\operatorname{sen}\theta = \frac{y}{1} = y$

$\cos\theta = \frac{x}{1} = x$

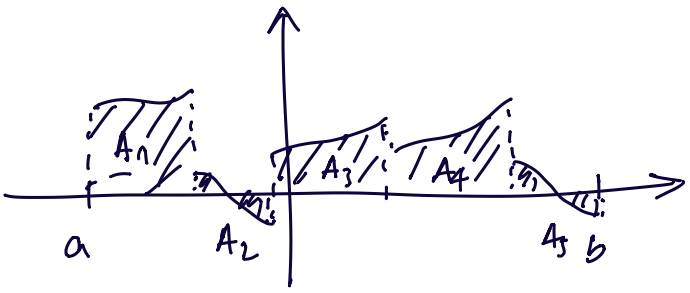


$\cos\pi = -1$

$\operatorname{sen}\pi = 0$



Teorema: Se f tem um número finito de saltos, então f é integrável.



$$\int_a^b f(x)\,dx = A_1 + A_2 + A_3 + A_4 + A_5$$

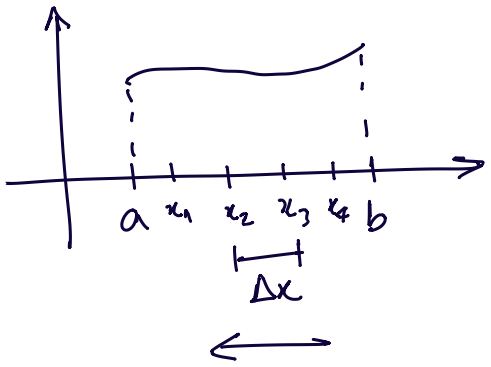
Propriedades:

1) $\displaystyle\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$ (limita)

2) $\displaystyle\int_a^b c \cdot f(x)\,dx = c \int_a^b f(x)\,dx$ (limita)

3) $\int_a^a f(x)\,dx = 0 \qquad (\Delta x = 0)$

4) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

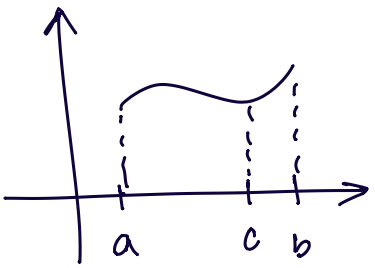


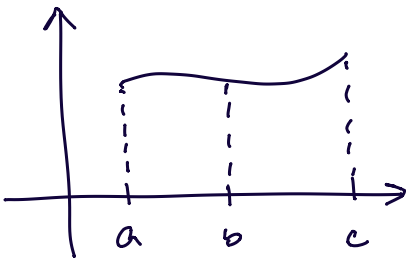
$\Delta x = x_i - x_{i-1} > 0 \Longrightarrow$

$\Delta x = x_{i-1} - x_i < 0 \Longleftarrow$

$\sum_{i=1}^{n} f(x_i^*)\cdot \Delta x$

5) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$





$\int_a^c f\,dx + \int_c^b f\,dx$

$= \int_a^c f\,dx - \int_b^c f\,dx$

$= \int_a^b f\,dx$

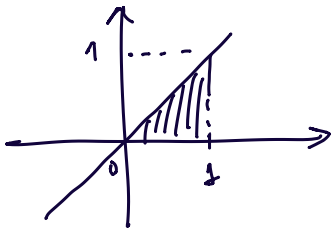
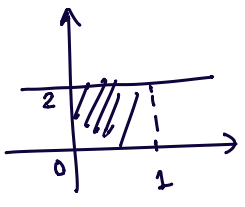
Exemplos: 1) $\int_0^1 4x^2\,dx = 4\int_0^1 x^2\,dx = 4\cdot\frac{1}{3} = \frac{4}{3}$

2) $\int_0^1 x^2 + 3x + 2\,dx = \int_0^1 x^2\,dx + 3\int_0^1 x\,dx + \int_0^1 2\,dx = \frac{1}{3} + 3\cdot\frac{1}{2} + 2$

$= \frac{11}{6} + 2$

$= \frac{23}{6}.$

$\int_0^1 x\,dx = \frac{1\cdot 1}{2} = \frac{1}{2}$ ($f(x) = x$)



$\int_0^1 2\,dx = 1\cdot 2 = 2$ ($f(x) = 2$)