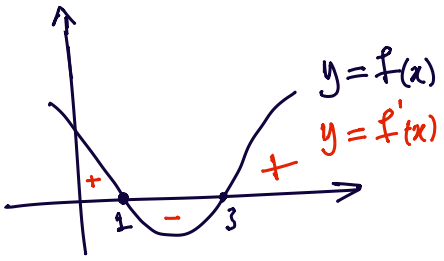
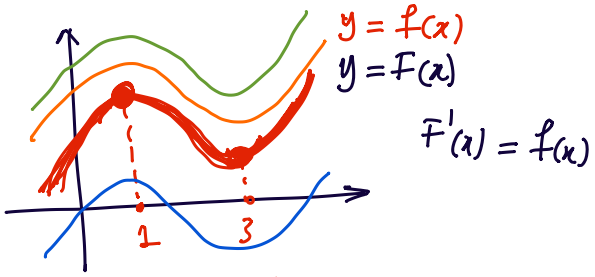
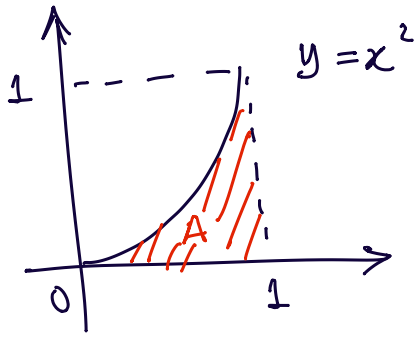


$f'(1) = 0$, $f'(3) = 0$ são ptos críticos candidatos a máx. ou mín.

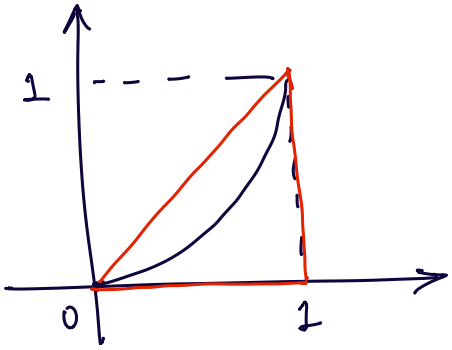


Como f' muda de sinal positivo p/ negativo ao redor de $x=1$, então $x=1$ é pto de máx. local. Analogamente, $x=3$ é pto de mín. local.

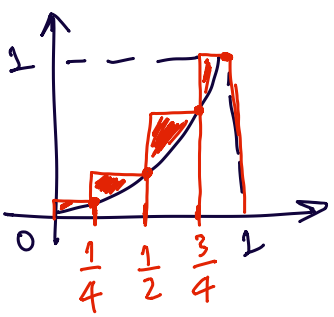
O problema da área



A = área entre o eixo x, o gráfico de f e $x=0$ e $x=1$.



$$A_{triâng.} = \frac{1 \cdot 1}{2} = \frac{1}{2} > A > 0$$



$$A_1 = \frac{1}{4} \cdot f\left(\frac{1}{4}\right) = \frac{1}{4} \cdot \left(\frac{1}{4}\right)^2 = \frac{1}{4} \cdot \frac{1}{16} = \frac{1}{64}$$

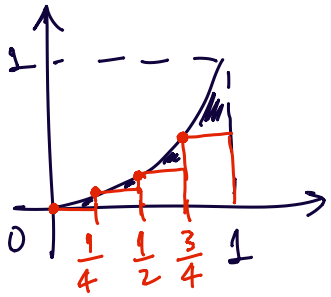
$$A_2 = \frac{1}{4} \cdot f\left(\frac{1}{2}\right) = \frac{1}{4} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$A_3 = \frac{1}{4} \cdot f\left(\frac{3}{4}\right) = \frac{1}{4} \cdot \left(\frac{3}{4}\right)^2 = \frac{1}{4} \cdot \frac{9}{16} = \frac{9}{64}$$

$$A_4 = \frac{1}{4} \cdot f(1) = \frac{1}{4} \cdot 1^2 = \frac{1}{4} \cdot 1 = \frac{1}{4}$$

$$A < A_1 + A_2 + A_3 + A_4 = \frac{1}{64} + \frac{1}{16} + \frac{9}{64} + \frac{1}{4} = \frac{1+4+9+16}{64}$$

$$= \frac{30}{64} = \frac{15}{32} = 0{,}46875.$$



$$A_1 = \frac{1}{4} \cdot f(0) = \frac{1}{4} \cdot 0 = 0$$

$$A_2 = \frac{1}{4} \cdot f\left(\frac{1}{4}\right) = \frac{1}{4} \cdot \frac{1}{16} = \frac{1}{64}$$

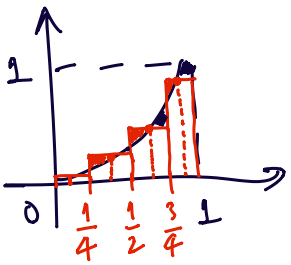
$$A_3 = \frac{1}{4} \cdot f\left(\frac{1}{2}\right) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$A_4 = \frac{1}{4} \cdot f\left(\frac{3}{4}\right) = \frac{1}{4} \cdot \frac{9}{16} = \frac{9}{64}$$

$$A > A_1 + A_2 + A_3 + A_4 = 0 + \frac{1}{64} + \frac{1}{16} + \frac{9}{64} = \frac{1+4+9}{64}$$

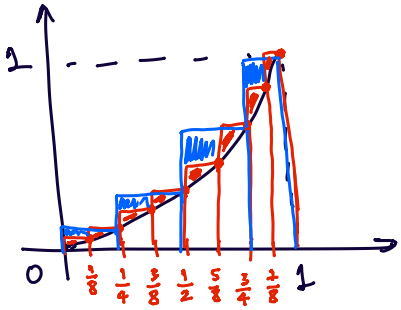
$$= \frac{14}{64} = \frac{7}{32} = 0{,}21875$$

$$0{,}21875 < A < 0{,}46875$$



Exercício:
Qual o valor da soma das áreas dos retângulos?

Aumentando o número de retângulos:



De modo geral,

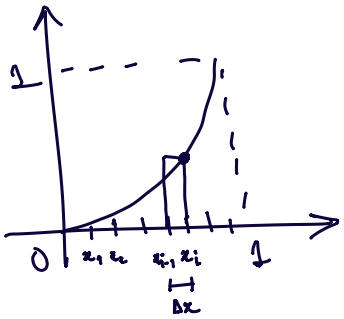
$$A \approx \sum_{i=1}^{n} A_i$$ (soma de Riemann)

Definimos

$$\boxed{A = \lim_{n\to\infty} \sum_{i=1}^{n} A_i}$$

quando o limite existe.

Calculando A:



$x_i =$

- particionando o intervalo $[0,1]$ em n partes iguais: $\Delta x = \frac{1}{n}$.
- tomando a altura dos retângulos usando o extremo direito de subintervalo.

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} A_i = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x \cdot f\left(\frac{i}{n}\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \cdot \left(\frac{i}{n}\right)^2$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \cdot \frac{i^2}{n^2} = \lim_{n\to\infty} \frac{1}{n^3} \sum_{i=1}^{n} i^2 = \lim_{n\to\infty} \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n\to\infty} \frac{2n^2+n+2n+1}{6n^2} = \lim_{n\to\infty} \frac{2n^2+3n+1}{6n^2}$$

$$= \lim_{n\to\infty} \frac{2n^2+3n+1}{6n^2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{2+\frac{3}{n}+\frac{1}{n^2}}{6} = \frac{2}{6} = \frac{1}{3}.$$

Definição: A integral definida de f no intervalo $[a,b]$ é definida por

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x \cdot f(x_i)$$

quando o limite existe.

Exemplo: $\int_0^1 x^2\,dx = \frac{1}{3}$.