$V, +, \cdot$, 10 propriedades.

$v + \vec{0} = v$, $v + (-v) = \vec{0}$

$\mathbb{R}^n$, $M(m \times n)$, $P_n(x)$, $F(x)$

$(a,b,c,d) \longleftrightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$(0,0,0,0) \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$(-a,-b,-c,-d) \quad \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$

$(1,2,3,4) \longleftrightarrow \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\dim(\mathbb{R}^4) = 4 \qquad \dim(M(2\times 2)) = 4$

$\{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$

$\left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$

$p(x) = a + bx + cx^2 + dx^3 \in P_3(x)$

$\dim(P_3(x)) = 4$

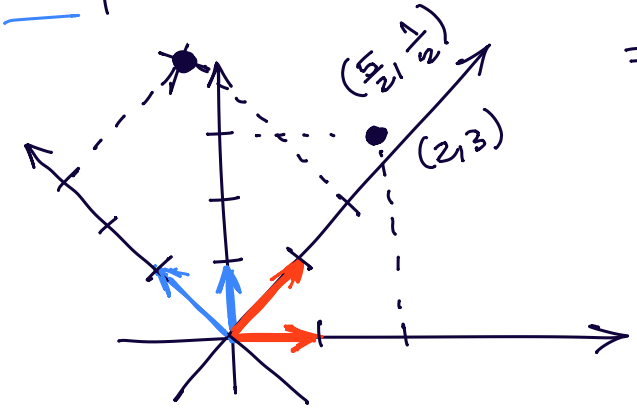
$\{1, x, x^2, x^3\}$

$1 + 0x + 0x^2 + 0x^3 \qquad 0 + 1x + 0x^2 + 0x^3$

$\{(1,0),(0,1)\} \subset \mathbb{R}^2$, $(2,3) = 2(1,0) + 3(0,1)$

$\{(1,1),(-1,1)\} \subset \mathbb{R}^2$, $(2,3) = a(1,1) + b(-1,1)$

$= (a-b, a+b)$



$\begin{cases} a - b = 2 \\ a + b = 3 \end{cases} \Rightarrow \begin{cases} a = 2 + b \\ a = 3 - b \end{cases}$

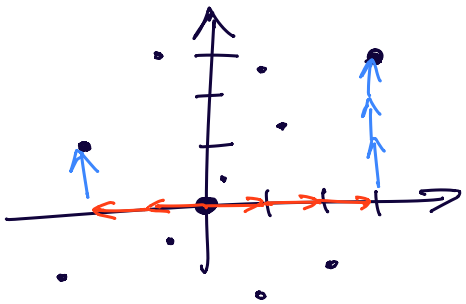
$2 + b = 3 - b \Rightarrow 2b = 1 \Rightarrow b = \frac{1}{2}$

$\Rightarrow a = 2 + \frac{1}{2} \Rightarrow a = \frac{5}{2}$

$T : V \to W$

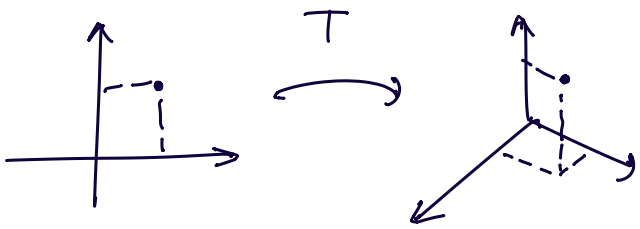
$T : \mathbb{R}^4 \to M(2\times 2)$

$T = ?$



Toda transformação linear é caracterizada por seu valor nos elementos da base do domínio.

$$T: \mathbb{R}^2 \to \mathbb{R}^3$$



$$\{(1,0),(0,1)\} \subset \mathbb{R}^2$$

$$T(1,0) = (1,2,3)$$

$$T(0,1) = (3,4,5)$$

$$T(x,y) = T\left(x(1,0)+y(0,1)\right) = T\left(x(1,0)\right) + T\left(y(0,1)\right)$$

$$= xT(1,0) + yT(0,1) = x(1,2,3) + y(3,4,5)$$

$$= (x+3y,\ 2x+4y,\ 3x+5y)$$

$$[T]_{3\times 2} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1a+0b \\ 1c+0d \\ 1e+0f \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \begin{matrix} a=1 \\ c=2 \\ e=3 \end{matrix}$$

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 0a+1b \\ 0c+1d \\ 0e+1f \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \Rightarrow \begin{matrix} b=3 \\ d=4 \\ f=5 \end{matrix}$$

$$\therefore [T] = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 3 & 5 \end{bmatrix}$$

2) $T: \mathbb{R}^2 \to \mathbb{R}^3$

$\{(1,1),(-1,1)\} \subset \mathbb{R}^2$

$T(1,1) = (1,2,3)$

$T(-1,1) = (3,4,5)$

$$[T]_{3\times 2} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} a+b \\ c+d \\ e+f \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \begin{cases} a+b=1 & ① \\ c+d=2 & ② \\ e+f=3 & ③ \end{cases}$$

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} -a+b \\ -c+d \\ -e+f \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \Rightarrow \begin{cases} -a+b=3 & ④ \\ -c+d=4 & ⑤ \\ -e+f=5 & ⑥ \end{cases}$$

① + ④ $\Rightarrow 2b = 4 \Rightarrow b = 2$ $\overset{①}{\Rightarrow} a+2=1 \Rightarrow a=-1$

② + ⑤ $\Rightarrow 2d = 6 \Rightarrow d = 3$ $\overset{②}{\Rightarrow} c+3=2 \Rightarrow c=-1$

③ + ⑥ $\Rightarrow 2f = 8 \Rightarrow f = 4$ $\overset{③}{\Rightarrow} e+4=3 \Rightarrow e=-1$

$$\therefore [T] = \begin{bmatrix} -1 & 2 \\ -1 & 3 \\ -1 & 4 \end{bmatrix}$$

$T(x,y) = (\,?\,,\,?\,,\,?\,)$

3) $T: \mathbb{R}^3 \to \mathbb{R}^2$?

$T(1,0,0) = (1,1)$

$T(0,1,0) = (1,-1)$

$T(0,0,1) = (-1,1)$