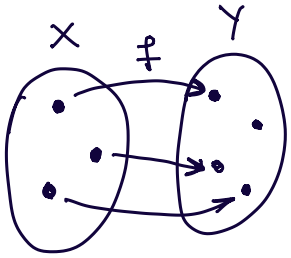
V espaço vetorial, $S = \{v_1, \dots, v_n\} \subset V$ base de V, $\dim V = n$

$u \in V$, $u = u_1 v_1 + \dots + u_n v_n \iff u = (u_1, \dots, u_n)$

$u_1, \dots, u_n$: coord. de u na base S

Função: X, Y não-vazios e f uma relação entre os elem. de X e Y tal que todo elem. de X está associado a um único elem. de Y.

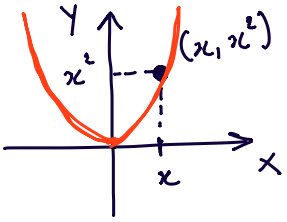


$f: X \to Y$

$x \mapsto y = f(x)$

$f: \mathbb{R} \to \mathbb{R}$

$x \mapsto x^2$



$T: \mathbb{R}^2 \to \mathbb{R}^3$

$(x,y) \mapsto T(x,y)$

$T(x,y) = (x+y+1, 2xy, x^2 - y^2)$

$T(0,0) = (1,0,0)$

$T(1,-1) = (1,-2,0)$

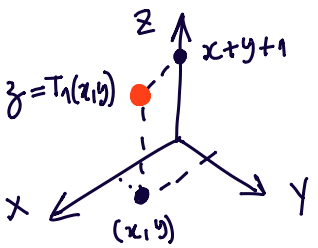
Observe que, se $T: \mathbb{R}^2 \to \mathbb{R}^3$, podemos decompor T

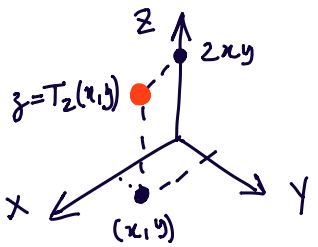
$T_1(x,y) = x+y+1$, $T_1: \mathbb{R}^2 \to \mathbb{R}$

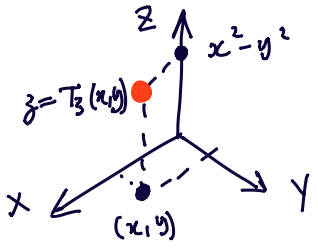
$T_2(x,y) = 2xy$, $T_2: \mathbb{R}^2 \to \mathbb{R}$

$T_3(x,y) = x^2 - y^2$, $T_2: \mathbb{R}^2 \to \mathbb{R}$

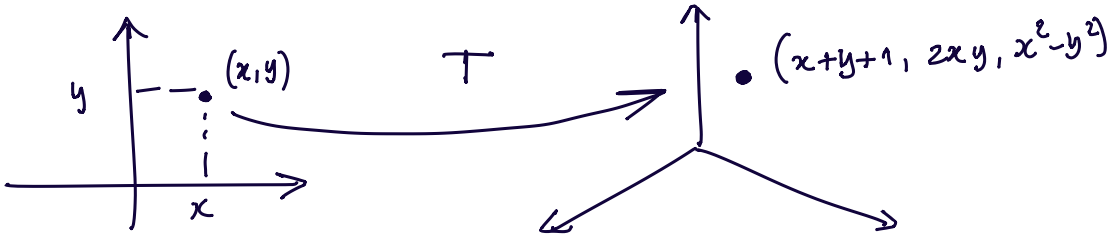
funções coordenadas de T







T:



Dizemos que $T: \mathbb{R}^n \to \mathbb{R}^m$ ($V \to W$) é uma <u>trasformação linear</u> se

i) $T(u+v) = T(u) + T(v)$, $\forall\, u, v \in \mathbb{R}^n$ (V)

ii) $T(\alpha u) = \alpha T(u)$, $u \in \mathbb{R}^n$ (V), $\alpha \in \mathbb{R}$.

<u>Exemplos</u>: 1) $T: \mathbb{R}^2 \to \mathbb{R}^6$, $T(x,y) = (x+y, x-y, x, y, 2x, 3y)$

$u = (x,y)$, $v = (a,b) \in \mathbb{R}^2$

$T(u+v) = T(x+a, y+b) = (x+a+y+b, x+a-y-b, x+a, y+b, 2x+2a, 3y+3b)$

$T(u) = T(x,y) = (x+y, x-y, x, y, 2x, 3y)$

$T(v) = T(a,b) = (a+b, a-b, a, b, 2a, 3b)$

$T(u) + T(v) = (x+y+a+b, x-y+a-b, x+a, y+b, 2x+2a, 3y+3b)$

$\therefore T(u+v) = T(u) + T(v)$

$T(\alpha u) = T(\alpha x, \alpha y) = (\alpha x + \alpha y, \alpha x - \alpha y, \alpha x, \alpha y, 2\alpha x, 3\alpha y)$

$\alpha T(u) = \alpha T(x,y) = \alpha(x+y, x-y, x, y, 2x, 3y) = (\alpha x + \alpha y, \alpha x - \alpha y, \alpha x, \alpha y, 2\alpha x, 3\alpha y)$

$\therefore T(\alpha u) = \alpha T(u)$

Portanto, T é uma transf. linear.

2) $T: \mathbb{R}^3 \to \mathbb{R}^2$ transf. linear, $S = \{\underbrace{(1,0,0)}_{e_1}, \underbrace{(0,1,0)}_{e_2}, \underbrace{(0,0,1)}_{e_3}\} \subset \mathbb{R}^3$

$T(1,0,0) = (a,b)$

$T(0,1,0) = (c,d)$

$T(0,0,1) = (e,f)$

a, b, c, d, e, f constantes

$u = (x,y,z) \in \mathbb{R}^3$

$u = (x,y,z) = x(1,0,0) + y(0,1,0) + z(0,0,1)$

$= x e_1 + y e_2 + z e_3$

$$T(x,y,z) = T(xe_1 + ye_2 + ze_3) = T(xe_1) + T(ye_2) + T(ze_3)$$

$$= xT(e_1) + yT(e_2) + zT(e_3)$$

$$= x(a,b) + y(c,d) + z(e,f)$$

$$= (ax, bx) + (cy, dy) + (ez, fz)$$

$$= (ax + cy + ez,\ bx + dy + fz) \in \mathbb{R}^2$$

Assim, se T é linear, suas funções coord. são comb. lineares das coord. do vetor u numa base do domínio.

$$T : \mathbb{R}^3 \to \mathbb{R}^2$$

$$T(x,y,z) = (a_{11}x + a_{12}y + a_{13}z,\ a_{21}x + a_{22}y + a_{23}z)$$

$$= \underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}}_{2\times 3} \cdot \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{3\times 1} = \underbrace{\begin{bmatrix} a_{11}x + a_{12}y + a_{13}z \\ a_{21}x + a_{22}y + a_{23}z \end{bmatrix}}_{2\times 1}$$

↖ matriz da transf. linear na base canônica

$$T : \mathbb{R}^3 \to \mathbb{R}^2 \quad , \quad A_{2\times 3}$$