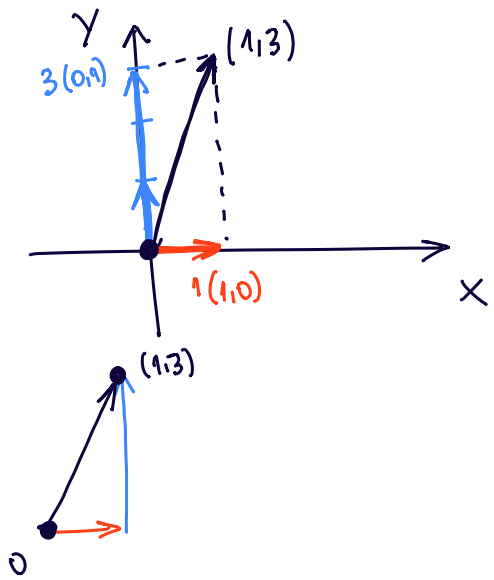
V, $S = \{v_1, v_2, \dots, v_n\} \subset V$, $v \in V \Rightarrow v = a_1 v_1 + \dots + a_n v_n$

S é LI e gera V, S é uma base.

$\dim V = \#S$, S é base.



$S = \{(1,0), (0,1)\} \subset \mathbb{R}^2$

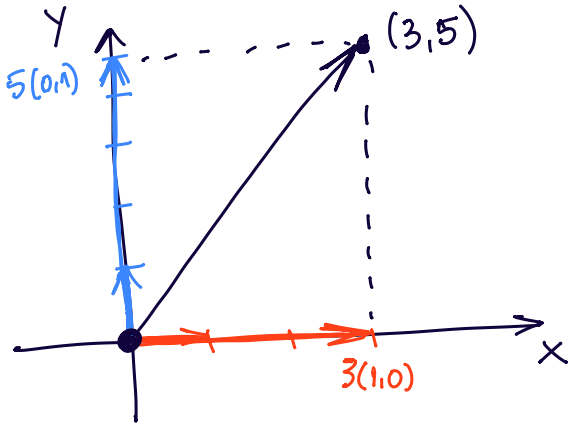
$(1,3) = \underset{1}{a}(1,0) + \underset{3}{b}(0,1)$

$= (a,0) + (0,b)$

$= (a,b)$

$\Rightarrow a = 1$, $b = 3$ ← coordenadas do vetor na base

$(3,5) = \underline{3}(1,0) + \underline{5}(0,1)$



$S = \{(1,1), (1,-1)\}$ é base de $\mathbb{R}^2$? ✓

- $a(1,1) + b(1,-1) = (0,0) \Rightarrow (a+b, a-b) = (0,0)$

$\Rightarrow \begin{cases} a+b = 0 \\ a-b = 0 \end{cases} \Rightarrow \begin{cases} a = -b \\ a = b \end{cases} \Rightarrow -b = b \Rightarrow 2b = 0 \Rightarrow \boxed{b = 0} \Rightarrow \boxed{a = 0}$

$\therefore$ S é LI

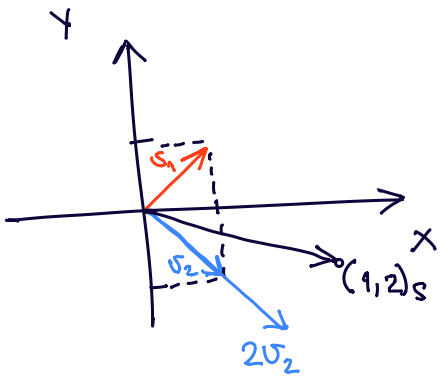
- Dado $(x_0, y_0) \in \mathbb{R}^2$

$$a(1,1) + b(1,-1) = (x_0, y_0) \Rightarrow \begin{cases} a+b = x_0 \\ a-b = y_0 \end{cases} \Rightarrow \begin{cases} a = x_0 - b \\ a = y_0 + b \end{cases}$$

$$\Rightarrow x_0 - b = y_0 + b \Rightarrow 2b = x_0 - y_0 \Rightarrow \boxed{b = \frac{x_0 - y_0}{2}}$$

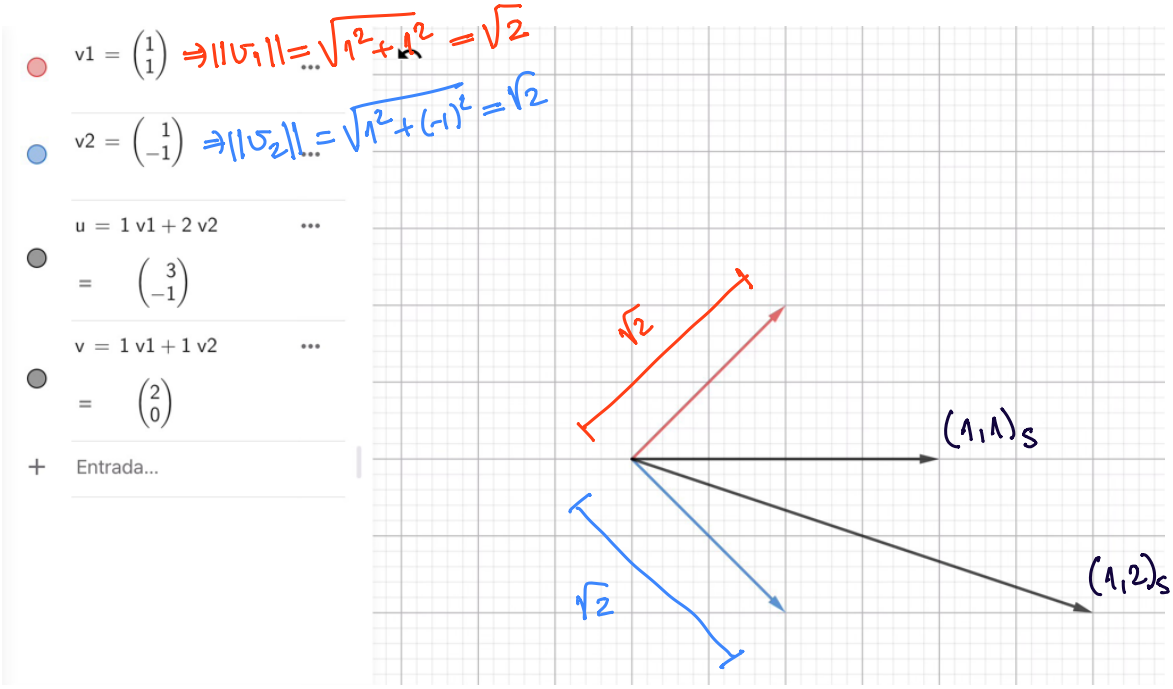
$$\Rightarrow a = y_0 + \frac{x_0 - y_0}{2} \Rightarrow a = \frac{2y_0 + x_0 - y_0}{2} \Rightarrow \boxed{a = \frac{y_0 + x_0}{2}}$$

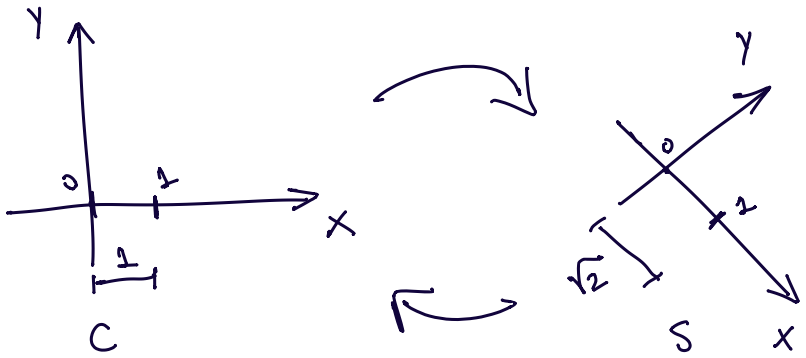
$\therefore$ S gera $\mathbb{R}^2$



$$S = \{\underbrace{(1,1)}_{v_1}, \underbrace{(1,-1)}_{v_2}\}, \quad C = \{(1,0),(0,1)\}$$

$$(1,2)_S = 1v_1 + 2v_2 \neq (1,2)_C$$





Como mudar de base?

$S = \{(1,1), (1,-1)\}$, $C = \{(1,0), (0,1)\}$

$(-1,2)_C = (a,b)_S$

$-1(1,0) + 2(0,1) = a(1,1) + b(1,-1)$

$(-1,2) = (a+b, a-b)$

$\Rightarrow \begin{cases} a+b = -1 \\ a-b = 2 \end{cases} \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = A^{-1}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

↖ fixa, independente do vetor dado na base canônica.

Analogamente, $(3,2)_S = (x,y)_C$

$3(1,1) + 2(1,-1) = x(1,0) + y(0,1)$

$(5,1) = (x,y)$

$(3\cdot 1 + 2\cdot 1, 3\cdot 1 + 2\cdot(-1)) = (x,y)$

$\Downarrow$

$\begin{cases} 1\cdot 3 + 1\cdot 2 = x \\ 1\cdot 3 - 1\cdot 2 = y \end{cases}$

$\Downarrow$

$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix}_S = \begin{bmatrix} x \\ y \end{bmatrix}_C$

$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}$

$|A| = -1 - 1 = -2$

$C \to S: \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}_C = \begin{bmatrix} \quad \end{bmatrix}_S$

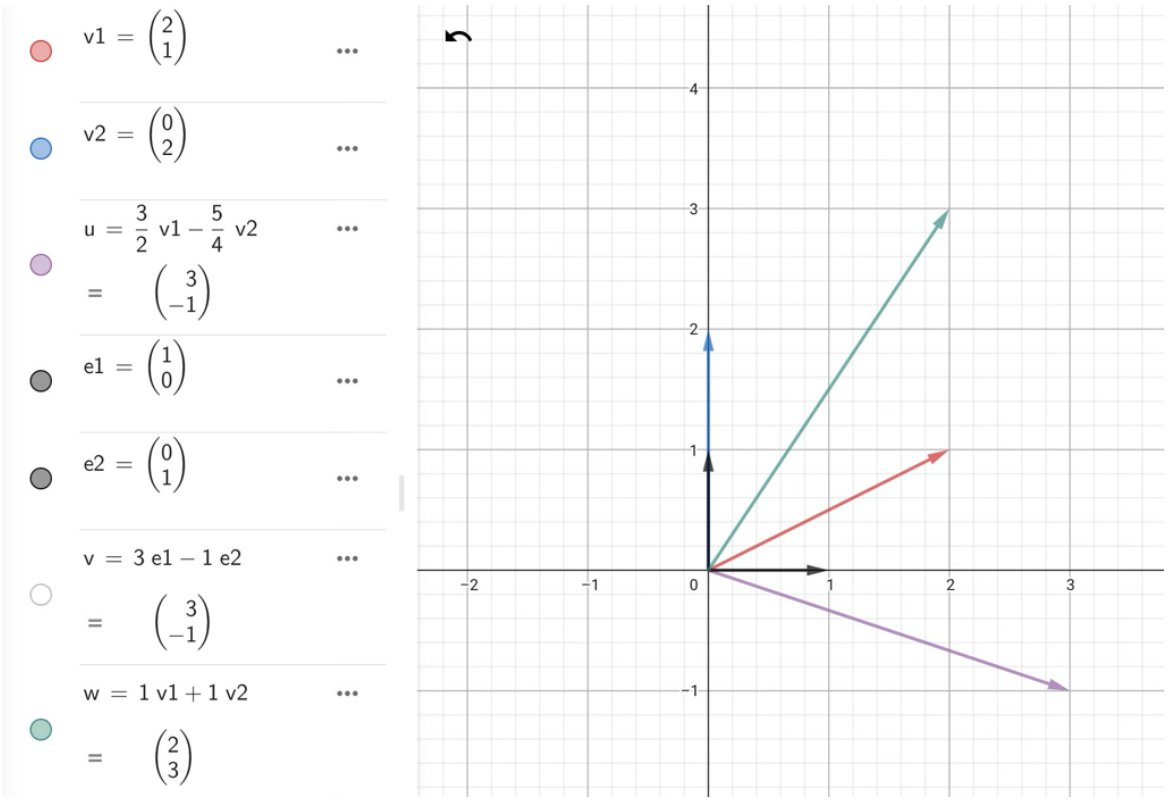
$S \to C: \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix}_S = \begin{bmatrix} \quad \end{bmatrix}_C$

→ matrizes de mudança de base

Exemplo: $S = \{(2,1),(0,2)\}$ é base de $\mathbb{R}^2$ (verifique)

$$(x,y)_C = a(2,1) + b(0,2) \Rightarrow \begin{cases} 2a = x \\ a + 2b = y \end{cases} \Rightarrow \underbrace{\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}}_{A} \begin{bmatrix} a \\ b \end{bmatrix}_S = \begin{bmatrix} x \\ y \end{bmatrix}_C \qquad S \to C$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ -1/4 & 1/2 \end{bmatrix} \qquad C \to S$$

$$(3,-1)_C = (a,b)_S : \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ -1/4 & 1/2 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 3/2 + 0 \\ -3/4 - 1/2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ -5/4 \end{bmatrix}$$

$$(1,1)_S = (x,y)_C : \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+0 \\ 1+2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$