UNIVERSIDADE FEDERAL DA GRANDE DOURADOS
Álgebra Linear
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(1) Determine as incógnitas em cada uma das igualdades de matrizes abaixo.

(a)
$$\begin{bmatrix} a-b & b+c \\ 2a+d & c-d \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 8 & 1 \end{bmatrix}.$$

(b)
$$\begin{bmatrix} x+y & 2x-z \\ y+w & z-w \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 5 & 2 \end{bmatrix}.$$

(c)
$$\begin{bmatrix} a+2b & c-d \\ a-c & 2b+d \end{bmatrix} = \begin{bmatrix} 9 & 2 \\ 3 & 7 \end{bmatrix}.$$

(d)
$$\begin{bmatrix} 2x-y & z+w \\ x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 5 & 1 \end{bmatrix}.$$

(e)
$$\begin{bmatrix} a-b+c & 2d \\ a+c & b-d \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 4 & 1 \end{bmatrix}.$$

(f)
$$\begin{bmatrix} x-2y & z+1 \\ 2x+w & y-z \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 10 & -2 \end{bmatrix}.$$

(g)
$$\begin{bmatrix} 2a+b & a-c \\ b+c & 3a-d \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 5 & 7 \end{bmatrix}.$$

(h)
$$\begin{bmatrix} x+y-z & 2w \\ x-z & y+w \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 1 & 5 \end{bmatrix}.$$

(i)
$$\begin{bmatrix} a+b & 2c-d \\ a-d & b+c \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 2 & 6 \end{bmatrix}.$$

(j)
$$\begin{bmatrix} x+2y & y-z \\ 2x-z & x+y+w \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 7 & 9 \end{bmatrix}.$$

(2) Usando as matrizes abaixo, resolva as operações indicadas, quando forem possíveis.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix},$$

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 2 & -1 & 1 \end{bmatrix}.$$

(a) $D + E$
(b) $D - E$
(c) $\mathrm{tr}(D)$
(d) A^T
(e) $2A^T + C$
(f) $B - B^T$
(g) $D^T + E^T$
(h) AB
(i) CA
(j) $D^T E$

(3) Calcule as potências indicadas das matrizes abaixo.

(a)
$$A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}.$$
Calcule A^2, A^3 e A^4.

(b)
$$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}.$$
Calcule A^2, A^3 e A^4.

(c)
$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$
Calcule A^2, A^3 e A^4.

(d)
$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$
Calcule A^2, A^3 e A^4.

(e)
$$A = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}.$$
Calcule A^2, A^3 e A^4.

(f)
$$A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}.$$
Calcule A^2, A^3 e A^4.

(g)
$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}.$$
Calcule A^2, A^3 e A^4.

(h)
$$A = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}.$$
Calcule A^2, A^3 e A^4.

(i)
$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$
Calcule A^2, A^3 e A^4.

j) $\begin{bmatrix}2&1\\1&2\end{bmatrix}\begin{bmatrix}2&1\\1&2\end{bmatrix}=\begin{bmatrix}4+1&2+2\\2+2&1+4\end{bmatrix}=\begin{bmatrix}5&4\\4&5\end{bmatrix}=A^2$

$A^3=A^2\cdot A=\begin{bmatrix}5&4\\4&5\end{bmatrix}\begin{bmatrix}2&1\\1&2\end{bmatrix}=\begin{bmatrix}14&13\\13&14\end{bmatrix}$

(j)

$$A=\begin{bmatrix}2&1\\1&2\end{bmatrix}.$$

Calcule A^2, A^3 e A^4.

(4) Determine os valores das incógnitas para que as matrizes abaixo sejam simétricas.

(a)

$$A=\begin{bmatrix}2&a&3\\4&5&b\\3&b&7\end{bmatrix}$$

$a_{ij}=a_{ji}$

$\begin{bmatrix}2&4&3\\4&5&0\\3&0&7\end{bmatrix}$ $a=4,\ b=b,\ b\in\mathbb{R}$

(b)

$$A=\begin{bmatrix}1&x&4\\y&2&z\\4&3&5\end{bmatrix}.$$

(c)

$$A=\begin{bmatrix}2&a+b&1\\a-b&5&c\\1&c&7\end{bmatrix}.$$

$a+b=a-b\Rightarrow b=-b\Rightarrow b=0$

$a\in\mathbb{R},\ c\in\mathbb{R},\ b=0$

(d)

$\begin{bmatrix}2&\pi+0&1\\\pi-0&5&\sqrt{2}\\1&\sqrt{2}&7\end{bmatrix}$

$$A=\begin{bmatrix}3&x+2&4\\2y&1&z\\4&z&5\end{bmatrix}.$$

(e)

$$A=\begin{bmatrix}1&a-2b&3\\4&2&c\\3&c&5\end{bmatrix}.$$

(f)

$$A=\begin{bmatrix}2&x+y&1\\x-y&4&z\\1&z&6\end{bmatrix}.$$

(g)

$$A=\begin{bmatrix}5&a&b+c\\2&3&d\\b+c&d&1\end{bmatrix}.$$

$a=2,\ b,c,d\in\mathbb{R}$

(h)

$$A=\begin{bmatrix}1&2a-b&4\\3&5&c\\4&c&2\end{bmatrix}.$$

$3=a+2b=-7+2\cdot 5$

$a=3-2\cdot 5$

$=-7$

(i)

$$A=\begin{bmatrix}4&x&y+1\\x&2&z\\y+1&z&3\end{bmatrix}.$$

(j)

$\begin{bmatrix}2&-3+2\cdot 3&-1\\3&5&0\\-1&0&8\end{bmatrix}$

$$A=\begin{bmatrix}2&a+2b&c\\3&5&d\\c&d&8\end{bmatrix}.$$

$c,d\in\mathbb{R}$

$a+2b=3\Rightarrow a=3-2b,\ b\in\mathbb{R}$

(5) Calcule a inversa das matrizes abaixo, quando existir.

(a)

$$A=\begin{bmatrix}1&1\\1&2\end{bmatrix}.$$

(b)
$$A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}.$$
(c)
$$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}.$$
(d)
$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}.$$
(e)
$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}.$$
(f)
$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$
(g)
$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$
(h)
$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 2 \\ 0 & 1 & 1 \end{bmatrix}.$$
(i)
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}.$$
(j)
$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}.$$

(6) Calcule os determinantes das matrizes abaixo.

(a)
$$\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix}.$$
(b)
$$\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}.$$
(c)
$$\begin{vmatrix} 3 & 5 \\ 2 & 7 \end{vmatrix}.$$
(d)
$$\begin{vmatrix} -1 & 4 \\ 2 & 3 \end{vmatrix}.$$
(e)
$$\begin{vmatrix} a & 2 \\ 3 & b \end{vmatrix}.$$

(f)
$$\begin{vmatrix} x & 1 \\ 2 & x \end{vmatrix}.$$
(g)
$$\begin{vmatrix} \lambda - 2 & 1 \\ 3 & \lambda + 1 \end{vmatrix}.$$
(h)
$$\begin{vmatrix} 2a & b \\ c & 3d \end{vmatrix}.$$
(i)
$$\begin{vmatrix} x + 1 & 2 \\ 3 & x - 1 \end{vmatrix}.$$
(j)
$$\begin{vmatrix} \lambda & 2 \\ -1 & \lambda - 3 \end{vmatrix}.$$

(7) Calcule os determinantes das matrizes abaixo usando o método dos cofatores.

(a)
$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 2 & 0 & 1 \end{vmatrix}.$$
(b)
$$\begin{vmatrix} 2 & 0 & 1 \\ 1 & 3 & 2 \\ 4 & 1 & 0 \end{vmatrix}.$$
(c)
$$\begin{vmatrix} -3 & 0 & 7 \\ 2 & 5 & 1 \\ -1 & 0 & 5 \end{vmatrix}.$$
(d)
$$\begin{vmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 4 & 3 \end{vmatrix}.$$
(e)
$$\begin{vmatrix} 2 & 1 & 0 \\ 0 & 3 & 4 \\ 1 & 2 & 5 \end{vmatrix}.$$
(f)
$$\begin{vmatrix} 1 & k & k^2 \\ 1 & k & k^2 \\ 1 & k & k^2 \end{vmatrix}.$$
(g)
$$\begin{vmatrix} x & 1 & 0 \\ 2 & x & 1 \\ 0 & 3 & x \end{vmatrix}.$$
(h)
$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 0 & 2 \end{vmatrix}.$$

(i)
$$\begin{vmatrix} 2 & -1 & 3 \\ 1 & 4 & 0 \\ 3 & 2 & 1 \end{vmatrix}.$$

(j)
$$\begin{vmatrix} 4 & 1 & 2 \\ 0 & 3 & 1 \\ 2 & 0 & 5 \end{vmatrix}.$$

(8) Encontre os valores de λ para os quais $\det(A) = 0$.

(a)
$$A = \begin{bmatrix} \lambda - 2 & 1 \\ -3 & \lambda + 1 \end{bmatrix}.$$

(b)
$$A = \begin{bmatrix} \lambda & 2 \\ 1 & \lambda - 3 \end{bmatrix}.$$

(c)
$$A = \begin{bmatrix} \lambda - 1 & 4 \\ 2 & \lambda + 2 \end{bmatrix}.$$

(d)
$$A = \begin{bmatrix} \lambda + 3 & 1 \\ 5 & \lambda - 2 \end{bmatrix}.$$

(e)
$$A = \begin{bmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{bmatrix}.$$

(f)
$$A = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda - 2 & 3 \\ 0 & 1 & \lambda + 1 \end{bmatrix}.$$

(g)
$$A = \begin{bmatrix} \lambda + 1 & 0 & 0 \\ 0 & \lambda - 3 & 2 \\ 0 & 1 & \lambda \end{bmatrix}.$$

(h)
$$A = \begin{bmatrix} \lambda - 2 & 1 & 0 \\ 0 & \lambda & 4 \\ 0 & 2 & \lambda - 1 \end{bmatrix}.$$

(i)
$$A = \begin{bmatrix} \lambda & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & \lambda \end{bmatrix}.$$

(j)
$$A = \begin{bmatrix} \lambda - 1 & 0 & 0 \\ 0 & \lambda + 2 & 1 \\ 0 & 2 & \lambda \end{bmatrix}.$$

(9) Resolva os sistemas lineares abaixo.

(a)
$$\begin{cases} x + y = 5, \\ 2x - y = 4. \end{cases}$$

(b)

$$\begin{cases} 2x + y = 7, \\ x - 3y = -5. \end{cases}$$

(c)

$$\begin{cases} x + y + z = 6, \\ 2x - y + z = 3, \\ x + 2y - z = 2. \end{cases}$$

(d)

$$\begin{cases} 2x + y - z = 1, \\ x - y + 2z = 4, \\ 3x + 2y + z = 7. \end{cases}$$

(e)

$$\begin{cases} x + 2y + z = 4, \\ 2x - y + 3z = 9, \\ x + y - z = 1. \end{cases}$$

(f)

$$\begin{cases} 2x - y + z = 5, \\ x + 2y - z = 1, \\ 3x + y = 7. \end{cases}$$

(g)

$$\begin{cases} x + 2y - z = 3, \\ 2x + 4y - 2z = 6, \\ x - y + z = 2. \end{cases}$$

(h)

$$\begin{cases} x + y + z = 2, \\ 2x + 2y + 2z = 5, \\ x - y = 1. \end{cases}$$

(i)

$$\begin{cases} x + 2y + z = 4, \\ 2x - y + z = 1, \\ x + y + 2z = 5. \end{cases}$$

(j)

$$\begin{cases} 2x + y - z = 3, \\ x - y + 2z = 4, \\ 3x + 2y + z = 8. \end{cases}$$

(10) Apenas observando as matrizes aumentadas abaixo, determine se o sistema correspondente possui solução e se a solução é única. Justifique sua resposta.

(a)

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \end{bmatrix}.$$

(b)

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$

(c)

inf. sol.

$$\begin{cases} x + 2z = 3 \\ y - z = 2 \\ 0 = 0 \end{cases}$$

(d)

$$\left[\begin{array}{ccc|c} x & y & z & \\ 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

posto(A|b) = 2
posto(A) = 2
< var. = 3

$$\left[\begin{array}{cccc} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right].$$

(e)

$$\left[\begin{array}{cccc} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

(f)

impossível

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 5 \end{array}\right].$$ ← 0 = 5

posto(A|b) = 3
posto(A) = 2

(g)

sol. única

$$\begin{cases} x = 2 \\ y = 3 \\ 0 = 0 \end{cases}$$

(h)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

posto(A|b) = 2
posto(A) = 2
= var = 2

$$\left[\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

(i)

$$\left[\begin{array}{cccc} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 4 \end{array}\right].$$

(j)

$$\left[\begin{array}{cccc} 1 & 2 & 0 & 3 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 1 & 5 \end{array}\right].$$

? exercício